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SYNCHRONIZATION OF ASYMPTOTICALLY PERIODIC BEHAVIORS IN COUNTABLE CELLULAR SYSTEMS

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Abstract. We address the question of frequencies locking in coupled differential systems and of the existence of (component) quasi-periodic solutions of some kind of differential systems. These systems named "cellular systems",are quite general as they deal with countable number of coupled systems in some general Banach spaces. Moreover, the inner dynamics of each subsystem does not have to be specified. We reach some general results about how the frequencies locking phenomenon is related to the structure of the coupling map, and therefore about the localization of a certain type of quasi-periodic solutions of differential systems that may be seen as cellular systems. This paper gives some explanations about how and why synchronized behaviors naturally occur in a wide variety of complex systems.

Keywords. Coupled systems, synchronization, frequencies locking, quasi-periodic motions, differential systems, asymptotically periodic.

1 Introduction

Synchronization is an extremely important and interesting emergent property of complex systems. The first example found in literature goes back to the 17th century with Christiaan Huygens' work [11, 2]. This kind of emergent behavior can be found in artificial systems as well as in natural ones and at many scales (from cell to whole ecological systems). Biology abounds with periodic and synchronized phenomena and the work of Ilya Prigogine shows that such behaviors arise within specific conditions: a dissipative structure generally associated to a nonlinear dynamics [20]. Biological systems are open, they evolve far from thermodynamic equilibrium and are subject to numerous regulating processes, leading to highly nonlinear dynamics. Therefore periodic behaviors appear (with or without synchronization) at any scale [21]. More generally, life itself is governed by circadian rhythms [9]. Those phenomena are as much attractive as they are often spectacular: from cicadia populations that appear spontaneously every ten or thirteen years [10] or networks of heart cells that beat together [17] to huge swarms in which fireflies, gathered in a same tree, flash simultaneously [3]. This synchronization phenomenon occupies a privileged position among emergent collective phenomena because of its various applications in neuroscience, ecology, earth Science, for instance [27, 25, 16], as well as in the field of coupled dynamical systems, especially through the notion of synchronization of chaotic systems [18, 7] and the study of coupled-oscillators [13]. This wide source of examples leads the field of research to be highly interdisciplinary, from pure theory to concrete applications and experimentations.

The classical concept of synchronization is related to the locking of the basic frequencies and instantaneous phases of regular oscillations. One of the most successful attempts to explore this emergent property is due to Kuramoto [14, 15]. As in Kuramoto's work, those questions are usually addressed by studying specific kinds of coupled systems (see for instance [5, 22, 8]). Using all the classical methods available in the field of dynamical systems, researchers study specific trajectories of those systems in order to get information on possible attracting synchronized state [28, 13, 22, 19, 8, 12].

The starting point of this work was the following question : "Why synchronization is such a widely present phenomena ?" In order to give some mathematical answer to this question, the first step is to build a model of coupled systems that is biologically inspired. This is done in the second section where, after having described some basic material, we define what we name cellular systems and cellular coupler. If one would summarize the specificities of cellular systems, one could say that each cell (subsystem) of a cellular system receives information from the whole population (the coupled system) according to some constraints:

- a cell has access to linear transformations of all the others cell's states
- the way this information is gathered depends (not linearly) on the cell's state itself

In other words, a cell interprets its own environment via the states of the whole population and according to its own state.

It is a bit surprising that despite this model arises very naturally, it gives a good framework to address the main

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