

# HARVESTING ON THE MATURE INDIVIDUALS OF A SINGLE POPULATION MODEL WITH IMPULSIVE TOXIN INPUT AND TIME DELAYS

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**Abstract.** This paper deals with the single stage-structured population model with impulsive toxin input and time delays which include individual maturation time delay and pollution time delay. What's more, the mature individuals are harvested continuously. We not only show that the population goes extinct if the harvesting rate is beyond a critical threshold, but also obtain conditions for exponentially asymptotic stability of population-extinction periodic solution and permanence of the population. In this case, we can be easy to control the harvesting rate, the impulsive period and amount of impulsive toxin input to protect the species. Our results contribute to biological resource management.

**Keywords.** Impulsive toxin input, Permanence; Stage-structured, Exponentially Asymptotic Stability.

## 1 Introduction

With the rapid development of modern industry and agriculture, the pollution of environment has become the one of the important problems in our lives, which has made many species extinction and others at the verge of extinction. Especially, the presence of toxicants in the environment decreases the growth rate of species. In recent years, using mathematical models-many researchers has studied effects of the pollutants on population(see[1, 2, 3, 4]) and some of them has studied the stage-structured models(see[5, 6, 7, 8, 9]). Most of their works assumed continuous input of toxin to the environment. But sometimes the toxins are emitted in regular pulses (see[10, 11]). For example, agricultural chemicals and some industrial waste are often emitted as pulses with regular intervals. As we known, generally, toxins don't have an influence on the population at once when they are emitted into the environment and it will last for some time before they can decrease the average growth rate of the species. So it is necessary to study the effect of pollution time delay on the extinction and permanence of population in a polluted environment. In this case, we can study the toxin modeling which includes

pulses and time delay instead of the previous continuous toxin modeling.

In the natural world, many species individual member's life can be classified into two distinct stages: immaturity and maturity. Some of the fact is that the mature individuals are affected by the toxin and the immature individuals are not. For example, tortoise is such a species whose eggs are laid on the beach and they are living in the water areas.

In this paper, according to the above biological background, we consider the harvesting on the mature population of the stage-structured population with impulsive toxin input and time delay in a polluted environment. We will obtain conditions for exponentially asymptotic stability of population-extinction periodic solution and permanence of the population. The results have an important meaning to control the environmental pollution and protect the ecological balance.

In section 2, we set up our model and some definitions and lemmas are given in section 3. In section 4, main results are proved and we give a brief conclusion of our results and some numerical simulations in section 5.

## 2 The Model

The following system was introduced by Aiello and Freedman (see[5]) as a model for single-species stage-structured dynamics

$$\begin{cases} \frac{dx_j(t)}{dt} = \alpha x(t) - d_j(t) - \alpha e^{-d\tau_1} x(t - \tau_1), \\ \frac{dx(t)}{dt} = \alpha e^{-d\tau_1} x(t - \tau_1) - \beta x^2(t). \end{cases} \quad (1)$$

where  $x_j(t)$  and  $x(t)$  represent the immature and mature population densities, respectively;  $\alpha$  is the coefficient of birth rate of the mature population in this environment;  $\tau_1$  is a constant;  $d$  is the coefficient of death rate of the immature population;  $\beta$  represents the intraspecific competition coefficient of mature population.

According to the above analysis, we consider not only the environmental toxin pulse input and the influence of pollution time delay on the population, but also harvesting of the mature population in a polluted environment

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