

# ON FORCED SYNCHRONIZATION OF A RING NETWORK OF HYPERCHAOTIC OSCILLATORS

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**Abstract.** This paper investigates, from a control theory viewpoint, the forced synchronization of a ring network of (controlled) hyperchaotic oscillators. In particular, regarding to an authentication process for secure communication or filtered access purposes, this paper considers a set of three continuous-time, hyperchaotic circuits of Matsumoto-Chua-Kobayashi [1] with unidirectional, partial-state coupling<sup>1</sup>, where one hyperchaotic (sub)system represents the “claimed identity” to be authenticated, while the two others can be viewed as locking/unlocking devices. The proposed authentication protocol is then based on performing a kind of tripartite *handshaking procedure*<sup>2</sup>, by means of synchronizing some state trajectories of interest (the authentication being validated if the synchronization is achieved). Thus, after briefly describing the authentication process, this paper focuses on transmitted states between each pair of nodes, with regard to confidentiality preserving criteria and the synchronization objective. Then, we address the forced synchronization problem, by considering both sliding mode controllers and endogenous control laws to drive each hyperchaotic subsystem. Finally, to illustrate the forced synchronization method, some simulation results are given with right and wrong “claimed identity”.

**Keywords.** Chaos synchronization, hyperchaos, chaotic networks, sliding-mode control, authentication.

## 1 Introduction

Since the seminal paper of Nijmeijer and Mareels [2], synchronization of chaotic systems with regard to a control theory viewpoint, has received an increasing interest (see, for instance, [3][4][5][6] and references therein). Such an attention is indeed motivated by potential applications of chaos synchronization to many fields (e.g. [7][8][9]), ranging from one pair of chaotic systems till large-scale networks with chaotic dynamics (e.g. [10][11][12][13]). This leads to numerous existing works devoted, for instance, to (private or secure) communication (e.g. [14][15]), chaos cryptography (e.g. [16][17][18]), and synchronization in chaotic networks (including some neural networks – e.g.

[19][20], some biological ones – e.g. [21], complex dynamical networks – e.g. [22][23][24][25][26], etc.). Following a somewhat different objective from those related to these two first concepts (that are secure communication and cryptography), this paper investigates the use of controlled synchronization of hyperchaotic systems<sup>3</sup> [27] as an authentication process (which can be used, for instance, for filtered access purposes). More precisely, we propose to consider the parameters and the mathematical structure of a hyperchaotic system as the signature of a “claimed identity” to be authenticated by means of an appropriate checking process. In this purpose, as suggested in [28], we investigate a kind of tripartite *handshaking procedure* based on the (controlled) synchronization properties of a ring network of unidirectionally coupled, hyperchaotic subsystems, including the “claimed identity” as one of them. With respect to such a context and chosen coupling conditions, the authentication is then validated if some selected state trajectories synchronize.

Such a controlled synchronization is nevertheless difficult to achieve due to:

- the hyperchaotic nature of each subsystem (as the high sensitivity of hyperchaotic systems to external signals intricate the control [29][30] and, therefore, the forced synchronization),
- the selected (partial-state) coupling between each pair of nodes (as this selection mainly comes from confidentiality preserving criteria rather than facilities for synchronization),

and,

- the fact that each control law (within each network node) may induce, through the coupling, some destabilizing effects on the following connected node.

According to that, this paper is organized as follows. Section 2 briefly describes the authentication process. Section 3 introduces, from a control theory viewpoint, the synchronization problem intrinsic to that process. Section 4 focuses on the ring network of hyperchaotic oscillators, including the choice of transmitted states between each

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<sup>1</sup>i.e. restricted to only one or two transmitted state(s) per network node, while the M-C-K circuit is a four-dimensional system.

<sup>2</sup>As in the context of secure communication protocols.

<sup>3</sup>That are chaotic systems with at least two positive Lyapunov exponents.