Journal of Nonlinear Systems and Applications (2010) 40–50

http://www.watam.org/JNSA/

THE TOPOLOGY OF TANK STIRRING

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Abstract. We study the topology of stirring in a cylindrical tank with a centrally located impeller and a recycle loop. In constrast to the usual approaches involving partial differential equations or statistical mechanics, we consider an idealized mathematical model which consists of a discrete time process. Our numerical studies indicate the existence of invariant tori, corresponding to periodic orbits, within which no mixing occurs. In addition to rigorously proving the existence of such tori, we prove that, under certain mild conditions, the topological structure which evolves is a "Sierpinski-like" curve $\times S^1$.
Keywords. cylindrical tank, recycle loop, stirring, mixing, invariant tori, Sierpinski curve.

1 Introduction

Our goal in this work is to study stirring in a tank from a new perspective. We construct a highly simplified model of tank stirring with a recycle loop, and then investigate the model both numerically and rigorously. We imagine a cylindrical tank into which two or more fluids of equal density have been placed. Stirring is done via the motion of an impeller with the goal of mixing the fluids completely, while fluid leaves the tank through the bottom and then returns through the top, as shown in Figure 1.

While in the tank fluid is stirred by an impeller fastened to a rod at the location of the axis of the tank. Chemical engineers know that such a tank will produce beautiful, but not "good" stirring. Segregated regions which persist and in which there is no mixing of fluids appear as a result of the impeller motion. In particular, there will always be two large toroidal segregated regions in the tank. One can stir forever, but these regions remain. See Figure 2. A better mixing strategy is the placement of the impeller off-center in the tank, as shown in Figure 3. Eccentrically placed impellers break the symmetry of motion and break up large segregated regions. However, we leave the study of eccentrically placed impellers for later work. In this paper, we wish to understand the dynamics and topology of structures occurring in the basic tank.

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‡Manuscript received April 19, 2009; revised January 11, 2010.

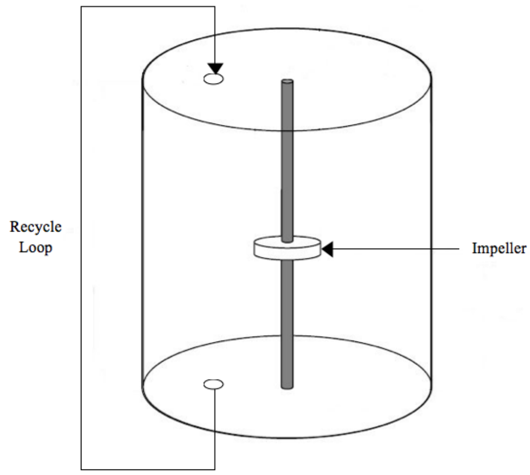

Figure 1: *A cylindrical tank with the impeller centered and a recycle loop, which moves fluid from the bottom of the tank to the top.*

The usual approaches to fluid flow have been through partial differential equations such as the Navier-Stokes equations or through statistical mechanics. Here we instead consider a discrete time process consisting of three maps from the tank to itself, and, also, in the cross section, from a rectangle in the plane to itself.

This work was initially motivated by two previous investigations. The first was a study of mixing in the chemical engineering dissertation of Justin Lacombe [4] from Rutgers University in 2006. Lacombe studied simulated mixing from a theoretical point of view, and studied it as a truly three-dimensional system. The model consisted of two independent flows on a solid torus: the first mimicked the behavior of the concentric laminar stirred tank, and the second introduced a perturbation. This model of Lagrangian mixing consisted of a system of differential equations, which were then integrated to give approximate solutions.

The second motivation was a paper entitled "The topology of stirred fluids" [3], which investigated simple idealized discrete models of 2D stirring. In the model, fluids were entering a chamber with overflow leaving it. The stirring created a Cantor-like but connected boundary between the fluids, and in many cases, it was shown that