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GENERATING SERIES : A COMBINATORIAL COMPUTATION

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Abstract. The purpose of this paper is to apply combinatorial techniques for computing coefficients of rational formal series (G_k) in two noncommuting variables and their differences at orders k and k-1.
This in turn may help one to study the reliability and the quality of a model for non-linear black-box identification.
We investigate the quality of the model throughout the criteria of a measure of convergence.
We provide, by a symbolic computation, a valuation relating to the convergence of the family (B_k). This computation is a sum of differential monomials in the input functions and behavior system. We identify each differential monomial with its colored multiplicity and analyse our computation in the light of the free differential calculus.
We propose also a combinatorial interpretation of coefficients of (G_k). These coefficients are powers of an operator Θ which is in the monoid generated by two linear differential operators Δ and Γ. More than a symbolic validation, these computing tools are parameterized by the input and the system's behavior.

Keywords. nonlinear systems, combinatorics, generating series, symbolic computation, model validation.

1 Introduction

Our topic is at the junction of several areas, area of identification and validation of non-linear systems and area of generating series in non commutative variables.

- The model validation is a crucial problem in system identification [9]. It measures confidence in the model to reproduce the behavior of a dynamic system, under some hypothesis.
  In a discrete-time approach, the model validation is really an invalidation since it determines whether a discret sample input-output is inconsistent with the model[10].
  In [11], the authors develop new methods for validation of continuous-time non-linear systems. They use Barrier certificates whose existence prove inconsistency of a model, with experimental data.
  To validate a continuous-time model of an unknown dynamic system [1], we propose an exact symbolic computation of coefficients of rational power series. We use a deterministic model (versus probabilistic one) by considering that data noises are bounded.

- Several methods may be used for determining the input-output behavior of a dynamical system : transfer functions, functional expansions (Volterra series) [14], and generating power series [8]. For single input systems, the transfert function can be used to find a linear approximation by means of Padé approximants [3].
  The formal power series in several noncommutative variables are efficient tools for dealing with functional expansions. The behavior of causal functionals [8, 7]is uniquely described by two noncommutative power series : the generating series and the Chen series. The Chen series measures the input contribution and is independent of the system.
  The generating series (i.e the Fliess'series) is the geometric contribution and it is independent of the input. This formal series in noncommutative variables is defined on the 'encoding alphabet' $Z = \{z_0, z_1, \cdots z_n\}$ corresponding to the system input. Then the output is obtained by interpreting words over Z like iterated integrals of inputs. Finally, we see that noncommutative algebra of formal power series is useful in differential geometry, to study the behavior and the structure of analytic dynamical systems.

Generating series in non commutative variables have been recently studied by several authors [4, 5, 6]. In [2], the authors consider the problem of computing the coefficients of rational series in two commutative variables. It is possibly a more challenging task to deal with the non commutative case.
So, the purpose of this paper is to apply combinatorial techniques for getting a close formula of coefficients of a rational formal series (G_k) in two noncommutative variables and their differences at orders k and k-1.
This in turn may help one to study validation of a family (B_k) of bilinear systems, described by the series (G_k) and global modeling of an unknown dynamical system (Σ).
Computing and bounding these differences, we propose an estimation of the error due to approximations by (B_k). This error computation is a sum of differential monomials in the input functions and behavior system. We identify

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