Journal of Nonlinear Systems and Applications (2010) 96–101

http://www.watam.org/JNSA/

SYNCHRONIZATION IN COMPLEX DYNAMICAL NETWORKS: A STATE OBSERVER BASED APPROACH

M. Boutayeb, A. Zemouche and J. J. Slotine *†‡

Abstract. We investigate an approach to synchronization design in complex dynamical networks in the case when only partial state coupling between nodes is available. The approach is based on state observers, and applies to systems with multi-dimensional links and possibly different output matrices. One of its main features is to provide a general solution to $N-1$ equality constraints. Synchronization in the dynamical network is then expressed in terms of exponential stability conditions deduced from contraction theory. The second main feature lies in the design and computation of the observers gain matrix through convex Linear Matrix Inequalities under non conservative conditions.
Keywords. networks, observer, synchronization.

1 Introduction

During the last decade, extensive research activities were devoted to study complex dynamical networks as models of many aspects of real-world systems. Indeed, interesting mathematical models such as small-world models or scale-free models [1],[2],[3],[4] were introduced recently to describe various dynamical systems in different area; without being exhaustive let us mention some of them, such as: Biological, Internet or Electrical Power network systems. The problem of synchronization become, therefore, a central question in particular to characterize behaviours of the dynamical networks. For doing so, several research works were recently devoted to analyze synchronization in complex network systems in different configurations such as connected neural networks with or without delays, synchronization of uncertain dynamical networks using robust impulsive techniques or synchronization in switching dynamical networks [5],[6],[7],[8],[9],[10],[11],[12] with extension to linear time varying systems [13]. Most of the obtained results assume the complete knowledge of the state vector coupling between two nodes. This condition is rarely satisfied and can be very expensive or impractical. As made precise in [14], synchronization can still be achieved if the internal dynamics of each node has suitable properties.

In the general case, however, when only one or few state components are available between any two nodes, one has to resort to more explicit constructions, such as the use of state observers. This is the approach we consider in this paper.

In the case of two chaotic master-slave systems, observers based approach for synchronization was largely investigated during the last two decades [15],[16],[17] with extensions to unknown input recovery [18],[19],[20]. However, very few works were developed to deal with more complex dynamical networks. In [21], Jiang et all. have proposed a simple and useful observer based approach for synchronization in complex dynamical networks, using one dimensional links between nodes. Using the same output matrix in the whole dynamical network is a key condition to write the global system into a simple and workable form.

Our contribution is inspired by [21] and may be seen as a generalization of their approach. Indeed, we consider the problem of synchronization in complex dynamical networks from a state observer point of view. The systems considered here are in the general form with multi-dimentional links and possibly different output matrices. This leads to a major difficulty, namely that (N-1) equality constraints should be solved. To cope with this problem we provide a parameterization technique to characterize the whole solutions. Stability is analyzed through the contraction theory [22],[14],[23], where the synthesis of the observers gain may be deduced from a non conservative Linear Matrix Inequality (LMI) condition [24]. To clarify this point, we consider synchronization of dynamical network with a general output matrix H_j (for node j, equation (16)) instead of using the same matrix H for all nodes (equation 15 and figure 2). This has the advantage to provide a large flexibility for communication between two nodes. Indeed, for many reasons, the transmitted variables (from node j to node i) are not necessary the same of the received ones as shown in figure 3. This configuration leads to solve the problem of synchronization in a general form

*M. Boutayeb and A. Zemouche are with Nancy University CRAN UMR 7039 - France . E-mails: jnsa@uwaterloo.ca, john@uwaterloo.ca

†J. J. Slotine is with MIT NSL Cambridge - USA. E-mail: david@uwaterloo.ca

‡Manuscript received October 5, 2009; revised June 11, 2010.