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DYNAMICAL BEHAVIORS, LINEAR FEEDBACK CONTROL AND SYNCHRONIZATION OF THE FRACTIONAL ORDER LIU SYSTEM

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Abstract. In this paper, some dynamical behaviors of the fractional order Liu system are investigated. It is found that chaos exists in this system with order less than 3. Chaos control and synchronization are achieved by using linear control technique. Simulation results are used to visualize and illustrate the effectiveness of the proposed control and synchronization methods.

Keywords. Fractional order Liu system, stability conditions, chaos, chaos control, synchronization, linear control technique.

1 Introduction

Although fractional calculus is such an old topic that it was introduced in the early 17th century, it has been extensively studied in the last decade by scientists, engineers and physicists [1-2].

There are many definitions of fractional order derivatives. The Riemann-Liouville definition of fractional derivatives [3] is given by

$$D^{\alpha} f(t) = \frac{d^l}{dt^l} J^{l-\alpha} f(t), \quad \alpha > 0, \tag{1}$$

where J^{θ} is the θ-order Riemann-Liouville integral operator which is given as

$$J^{\theta} u(t) = \frac{1}{\Gamma(\theta)} \int_0^t (t-\tau)^{\theta-1} u(\tau) d\tau, \qquad \theta > 0.$$

Another one is the Caputo definition of fractional derivatives [4], which is often used in real applications:

$$D_*^{\alpha} f(t) = J^{l-\alpha} f^{(l)}(t), \tag{2}$$

where $f^{(l)}$ represents the l-order derivative of $f(t)$ and $l = [\alpha]$, this means that l is the first integer which is not less than α. The operator D_*^{α} is called the "Caputo differential operator of order α". In [5], F. Ben Adda studied the geometric and physical interpretation of the fractional derivative.

On the other hand, studying chaos in fractional order dynamical systems is very interesting topic and has much increasing attention in the past few years. Chaotic attractors have been found in the following fractional order systems, Lorenz [6], Chua [7], Chen [8] and Lü [9]. Chaos in fractional order autonomous systems has also an interesting phenomenon, that is, it can occur for orders less than three and this can not happen in their integer order counterparts according to the Poincaré-Bendixon theorem. Chaos control and synchronization in integer order differential systems are well understood [10-11], but they are still in the beginning in the case of fractional order chaotic systems. Recently, many papers about chaos control and synchronization in fractional order chaotic systems have been published by authors [12-15].

In this work, I study stability, chaos, control and synchronization in the fractional order Liu system. I use some Routh-Hurwitz conditions to study the stability conditions in this system. It is found that the lowest order for chaos to exist in this system is 2.55. The sufficient conditions for chaos control are derived analytically using linear feedback control technique. Conditions for achieving chaos synchronization via linear control method are studied using the classical Laplace transformation theory. The analytical results are verified by numerical simulations.

2 The Liu system with fractional order

The chaotic Liu system [16] is described by the following equations:

$$\begin{aligned} \frac{dx}{dt} &= a(y-x), \\ \frac{dy}{dt} &= bx - kxz, \\ \frac{dz}{dt} &= -cz + hx^2, \end{aligned} \tag{3}$$

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