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HOMOTOPY ANALYSIS METHOD FOR FRACTIONAL DIFFUSION EQUATION WITH ABSORBENT TERM AND EXTERNAL FORCE

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Abstract. In this paper, the diffusion equation with fractional time derivative α $(0 < \alpha \leq 1)$ with external force and absorbent term is considered. Using initial condition, the approximate analytical solution of the problem is obtained. Selecting the proper values of auxiliary and homotopy parameters, the convergence of the approximate series solution is presented for two different particular cases which showcase the effectiveness and potential of the method.
Keywords. Partial differential equation; Diffusion equation; IVP's; Caputo derivative; Homotopy analysis method.

1 Introduction

In this article we focus our attention to find the solution of the fractional diffusion equation with external force and absorbent term

$$\frac{\partial^\beta u(x,t)}{\partial t^\beta} = \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial}{\partial x}(F(x)u(x,t)) - \int_0^t \alpha(t-\xi)u(x,\xi)d\xi, 0 < \alpha \leq 1 \quad (1)$$

with the initial condition

$$u(x,0) = f(x), \quad x > 0, t > 0 \quad (2)$$

Here $\alpha(t)$ is a time-dependent absorbent term which may be related to a reaction diffusion process. The diffusion equation have been widely studied due to various applications in physics and engineering but the study assumes a different direction when in the classical diffusion equation the time derivative is replaced by a fractional derivative of order β $(0 < \beta \leq 1)$. An important outcome of these evolution equations is that it generates fractional Brownian motion which is a generalization of Brownian motion. Schot et al. [1] has given an approximate solution of the equation with the absorbent term and a linear external force in terms of Fox H-function. Zahran [2] has offered a closed form solution in Fox H-function of the generalized fractional reaction-diffusion equation subjected to an external linear force field, one that is used to describe the transport processes in disorder systems. Recently Das and Gupta [3] have solved similar equations by homotopy perturbation method (HPM). Owing to the restricted applications of analytical methods and rounding of errors involved in numerical techniques used by the researchers, the authors have made a sincere effort to solve the fractional diffusion equation by using a powerful analytical method called Homotopy Analysis Method (HAM) proposed by Liao [4]. The method which is based on homotopy, a fundamental concept in topology and differential geometry is an analytical approach to get the series solution of linear and nonlinear partial differential equations (PDE's). The difference with the other perturbation methods is that this method is independent of small / large physical parameters. It also provides a simple way to ensure the convergence of a series solution. Moreover the method provides ample freedom in the selection of a base function to approximate the linear and nonlinear problems ([5], [6]). Another advantage of the method is that one can construct a continuous mapping of an initial guess approximation to the exact solution of the given problem through an auxiliary linear operator and ensure the convergence of the series solution with the auxiliary parameter being used. Recently, Liao [7] has claimed that the difference with the other analytical methods is that one can ensure the convergence of series solution by means of choosing a proper value of convergence-control parameter. The use of HAM can be found in the recent works of Jafari and Seifi ([8],[9]), Jafari et al. [10], Hashim et al. [11] for solving linear and nonlinear time fractional PDE's.

In this article more accurate, flexible and very powerful analytical method HAM is used to solve the equation (1). Using the initial condition (2), the approximate analytical solution of $u(x,t)$ is obtained. As a particular case of the problem, the solutions of $u(x,t)$ for the initial conditions $f(x) = x$ and x^2 are calculated for different fractional Brownian motions and also for standard motion in the presence of external force and absorbent term.

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