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DRAWING SOLUTION CURVE OF DIFFERENTIAL EQUATION *†

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Abstract. We develop a method for drawing solution curves of differential equations. This method is based on the juxtaposition of local approximating curves on successive intervals $[t_i, t_{i+1}]_{0\leq i\leq n-1}$. The differential equation, considered as a dynamical system, is described by its state equations and its initial value at time t_0.
A generic expression of its generating series G_t truncated at any order k, of the output and its derivatives $y^{(j)}(t)$ expanded at any order k, can be calculated. The output and its derivatives $y^{(j)}(t)$ are expressed in terms of the coefficients of the series G_t and of the Chen series.
At the initial point t_i of every interval, we specify the expressions of G_t and $y^{(j)}(t)$. Then we obtain an approximated output $y(t)$ at order k in every interval $[t_i, t_{i+1}]_{0\leq i\leq n-1}$.
We have developed a Maple package corresponding to the creation of the generic expression of G_t and $y^{(j)}(t)$ at order k and to the drawing of the local curves on every interval $[t_i, t_{i+1}]_{0\leq i\leq n-1}$.
For stable systems with oscillating output, or for unstable systems near the instability points, our method provides a suitable result when a Runge-Kutta method is wrong.
Keywords. Curve drawing, differential equation, symbolic algorithm, generating series, dynamical system, oscillating output

1 Introduction

The usual methods for drawing curves of differential equations consist in an iterative construction of isolated points (Runge-Kutta). Rather than calculate numerous successive approximate points $y(t_i)_{i\in I}$, it can be interesting to provide some few successive local curves $\{y(t)\}_{t\in[t_i,t_{i+1}]_{0\leq 1\leq n-1}}$.
Moreover, the computing of these local curves can be kept partly generic since a generic expression of the generating series G_{t_i} of the system can be provided in terms of t_i. The expression of the local curves $\{y(t)\}_{t\in[t_i,t_{i+1}]}$ is only a specification for $t = t_i$ at order k of the formula given in the proposition of section 3.
We consider a differential equation

$$y^{(N)}(t) = \phi(t, y(t), \cdots, y^{(N-1)}(t)) \qquad (1)$$

with initial conditions

$$y(0) = y_{0,0}, \cdots, y^{(N-1)}(0) = y_{0,N-1}$$

We assume that $\phi(t, y(t), \cdots, y^{(N-1)}(t))$ is polynomial in $y, \cdots, y^{(N-1)}$.
Then this differential equation can be viewed as an affine single input dynamical system.

2 Preliminaries

The generating series ([5, 6, 9]) of a nonlinear dynamical system is a useful tool for a lot of problems: identification of dynamical systems ([13, 15, 16]), realization ([17]), approximation of dynamical systems by bilinear ones ([11, 14]), study of the stability of bilinear systems ([1]) and application to diabete ([2, 3]), decomposition of dynamical system into subsystems ([10]).

In this section, we are interested by the generating series of dynamical affine system, used as an accurate tool for drawing its output, particularly when the system is unstable or when the output is oscillating.

2.1 Affine system, Generating series

We consider the nonlinear analytical system affine in the input:

$$(\Sigma) \qquad \left\{ \begin{array}{rcl} \dot{q} &=& f_0(q) + \sum_{j=1}^{m} f_j(q)u_j(t) \\ y(t) &=& g(q(t)) \end{array} \right. \qquad (2)$$

- $(f_j)_{0\leq j\leq m}$ being some analytical vector fields in a neighborhood of $q(0)$
- g being the observation function analytical in a neighborhood of $q(0)$

Its initial state is $q(0)$ at $t = 0$. The generating series G_0 is built on the alphabet $Z = \{z_0, z_1, \cdots, z_m\}$, z_0 coding the drift and z_j coding the input $u_j(t)$. Generally G_0 is expressed as a formal sum $G_0 = \sum_{w\in Z^*} \langle G_0|w\rangle w$ where $\langle G_0|z_{j_0}\cdots z_{j_l}\rangle = f_{j_0}\cdots f_{j_l}g(q)|_{q(0)}$ depends on $q(0)$.

2.2 Fliess's formula and iterated integrals

The output $y(t)$ is given by the Fliess's equation ([6]):

$$y(t) = \sum_{w\in Z^*} \langle G_0|w\rangle \int_0^t \delta(w) \qquad (3)$$

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