Journal of Nonlinear Systems and Applications (2011) 122–133

http://www.watam.org/JNSA/

CLASSICAL STABILITY THEORY EXTENDED TO HYBRID DYNAMICAL SYSTEMS

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Abstract. Hybrid systems possess dynamical properties that combine the features of classical continuous-time systems and discrete-time systems. Consequently, it is natural to ask whether classical stability concepts and analysis tools apply to hybrid systems. Based on existing results in the literature, this article summarizes the degree to which the answer to this question is "yes". In fact, it indicates that most of the stability analysis tools for classical nonlinear systems apply to hybrid systems. This includes Lyapunov functions that decrease along solutions, relaxed Lyapunov conditions based on an invariance principle, and several tools that are a result of robustness of asymptotic stability, including Jacobian linearization, cascades and other reduction principles, results for slowly-varying systems, and averaging and singular perturbation theory. These tools are illustrated with examples or else references are given where more information can be found.

Keywords. Hybrid systems, asymptotic stability, Lyapunov functions, robustness, input-to-state stability.

1 Introduction

A good strategy for learning something new is to tie it mentally to something familiar. For example, consider embarking on a study of stability theory for nonlinear systems. In this case, it is helpful to view assessing stability for linear systems, either for continuous-time systems $\dot{x} = Ax$ or discrete-time systems $x^+ = Ax$, as the problem of finding symmetric, positive definite matrices P and Q such that $A^T P + PA = -Q$, for continuous-time systems, or $A^T PA - P = -Q$ for discrete-time systems. Indeed, the matrix P leads to a Lyapunov function $V(x) = x^T Px$ that decreases along solutions, and it is this property, rather than any generalization of eigenvalues of the matrix A, that is used to guarantee asymptotic stability in nonlinear systems. After a short study of nonlinear systems and a search for their Lyapunov functions, a student quickly awakens to the reality that Lyapunov functions are much harder to find for nonlinear systems and that additional tools, which are not as important for linear systems, should be considered. This observation leads to studying approaches like relaxed Lyapunov conditions based on the invariance principle, Jacobian linearization, stability when parameters vary slowly, stability of cascaded systems and other reduction principles, averaging and singular perturbation theory, and analysis tools based on dissipativity theory and input-to-state stability.

Hybrid systems constitute an extension of continuous-time and discrete-time nonlinear systems. In particular, they combine these two types of systems. When setting out to understand stability theory for hybrid systems, it is reasonable to start by asking about the degree to which stability analysis tools for nonlinear systems extend to hybrid systems. If not many of these tools extend, then we may be led to new tools. The purpose of this article is to point out that most, if not all, of the classical stability analysis tools for nonlinear systems extend to hybrid systems. Because of this fact, we leave to other articles the development for hybrid systems of stability analysis tools that have no analogue in classical stability analysis theory.

This article summarizes some of the contributions of the author and his colleagues over the last decade in the area of stability theory for hybrid systems. The summary is not exhaustive; rather, it features the highlights. The reader is directed to the tutorial article [22] for more details and examples, as well as an extensive list of references, including references to differing points of view about hybrid systems.

2 Hybrid Dynamical Systems

2.1 Their nature

Hybrid systems combine continuous-time and discrete-time dynamics. In particular, the state of a hybrid system can change continuously at certain locations in the state space and can change instantaneously in other (not necessarily different) locations in the state space. Continuous evolution is referred to here as "flow" while instantaneous change is called a "jump". Hybrid systems may contain continuous-valued variables, like positions and velocities as well as timers, and may also contain discrete-valued variables, like counters and logic states used to indicate conditions like "on", "off", "red", and "green", although these states are typically associated with integer values and the state is embedded in a finite-dimensional Euclidean space. Hybrid dynamical systems

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