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ON PERMANENCE AND STABILITY OF A LOGISTIC MODEL WITH HARVESTING AND A CARRYING CAPACITY DEPENDENT DIFFUSION

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Abstract. For the reaction-diffusion equation describing the harvested population with the logistic type of growth and diffusion stipulated by the carrying capacity K

$$\frac{\partial u(t,x)}{\partial t} = D\Delta\left(\frac{u(t,x)}{K(x)}\right) + r(x)u(t,x)\left(1 - \frac{u(t,x)}{K(x)}\right) - E(x)u(x,t)$$

existence, positivity, persistence and stability of solutions is investigated. In the logistic model the introduction of the standard diffusion term Δu (incorporated with the zero Neumann boundary conditions) leads to the situation when the population tends to be equally distributed over the space available, even if the carrying capacity $K(x)$ varies significantly with location. A K-driven diffusion was introduced to account for this effect. Harvesting with a prescribed revenue is also considered, which can lead to more than one possible solution for the harvesting effort.

Keywords. Logistic equation, diffusion, harvesting, revenue, positive periodic solution, global attractivity, rate of convergence.

1 Introduction

Population biology is one of significant and dynamic areas of mathematical biology, while sustainable harvesting and optimal resources management are among the most important problems of population ecology [9]. If the model describing population growth encounters spatial distribution and population diffusion, in addition to harvesting, the analysis of such systems becomes more complicated. However, models with diffusion more adequately describe management of spatially distributed resources, which is the reason why they recently attracted a lot of attention. Among other issues, optimal harvesting and stability of solutions were investigated [1, 2, 3] and how stability properties are influenced by diffusion [16].

The solution of the diffusive logistic model

$$\frac{\partial u(t,x)}{\partial t} = D\Delta u(t,x) + r(x)u(t,x)\left(1 - \frac{u(t,x)}{K(x)}\right), \quad t > 0, \quad x \in \Omega, \tag{1.1}$$

where $u(t,x)$ is the population density, $r(x)$ is the intrinsic growth rate, $K(x)$ is the carrying capacity of the environment (generally, both can also be time-dependent), with the zero Neumann boundary condition and a high diffusion rate D tends to be uniformly distributed. This assumption is reasonable for spatially uniform carrying capacity (K is not x-dependent), but for nonuniform resources distribution model (1.1) suggests that species may move to the regions with lower per capita available resources which doesn't seem to be biologically feasible. Diffusive systems of type (1.1) were considered in many papers, see, for example, [2, 3, 4, 6, 8, 12, 13, 15, 19, 20] and references therein.

We assume an alternative type of diffusion when not u but u/K diffuses, which is described by the equation

$$\frac{\partial u(t,x)}{\partial t} = D\Delta\left(\frac{u(t,x)}{K(x)}\right) + r(x)u(t,x)\left(1 - \frac{u(t,x)}{K(x)}\right), \quad t > 0, \quad x \in \Omega, \tag{1.2}$$

with the Neumann boundary condition

$$\frac{\partial\left(\frac{u}{K}\right)}{\partial n} = 0, \ x \in \partial\Omega, \ t \in (0,\infty).$$

This means that for very high diffusion the population tends to have not a uniform distribution over the domain but the uniform per capita available resources (u/K is constant).

This model was first introduced in [5], its stability properties (also in the case of time-dependent K and r) were studied in [14]. The optimal harvesting of (1.2) was investigated in [5], but not existence and stability issues for the harvested model.

In the present paper we study the generalization of model (1.2) to the case of harvested populations when the term of type $qE(x)u(t,x)$ is subtracted from the right-hand side. Here $E(x)$ is the harvesting effort, q characterizes either catchability ($0 < q \le 1$) per unit effort or both by-catch mortality and harvesting incorporated in the harvesting event. We will assume that in the former case per capita catchability never exceeds the effort applied, while in the latter case the effect of harvesting (which can

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