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CONDITIONS FOR QUADRATIC STABILITY OF A MULTI-PARAMETER SWITCHING SYSTEM

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Abstract. Necessary and sufficient conditions are obtained for the existence of a common quadratic Lyapunov function for a family of non-commuting matrices. Each of the matrices in the family is obtained from a fixed stable matrix by adding a sum of rank one matrices with positive coefficients. The conditions obtained are algebraic and involve checking positivity of matrices formed from the resolvent of the original stable matrix. The conditions can be viewed as a multi-parameter generalization of the Kalman-Yacubovich-Popov Lemma.

Keywords. Quadratic Lyapunov function, switching system, closedness of operator image, dual cone

1 Introduction and statement of results

The question of stability for switching systems has attracted much interest in recent years, and has led to a number of approaches and results (see [8] for a recent review). The focus of this paper is on the question of quadratic stability, meaning the existence of a quadratic Lyapunov function for the switched system. Quadratic stability is a robust property as it implies stability under arbitrary switching sequences. It provides a sufficient but not necessary condition for stability, so there are systems which fail to be quadratically stable but which maintain stability under arbitrary switching. Nevertheless quadratic stability is useful because it provides an obvious first filter for testing stability, and as we show in this paper there are cases where it can be checked using algebraic conditions. The results below are stated for linear switched systems, but they have extensions to nonlinear systems which can be dominated by stable linear systems.

We now recall the definition of quadratic stability. A collection of matrices

$$\{A_s \in \mathbb{R}^{n\times n} \,:\, s \in \mathcal{I}\} \tag{1}$$

(where $\mathcal{I}$ is an index set, possibly infinite) is said to have a common quadratic Lyapunov function (CQLF) if there is a positive definite matrix $P = P^T > 0$ such that

$$P\,A_s + A_s^T\,P < 0 \quad \text{for all } s \in \mathcal{I} \tag{2}$$

As discussed above, this condition is important because it guarantees exponential stability of the switched dynamical system

$$\frac{dx}{dt} = A_{s(t)}\,x, \quad s : [0,\infty) \to \mathcal{I} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system at time t, and s is any piecewise continuous function. Namely, using the quadratic Lyapunov function $V(x) = x^T P x$, it follows from (2), (3) that $dV(x(t))/dt < 0$ for all t.

Such switching systems arise in many areas of applications and finding conditions for stability is an important problem. For a general collection of matrices there is no known method to check the condition (2) other than by performing a numerical search, although there are special classes of matrices where other algebraic and analytic methods are available (see for example the cases described in [8]). One disadvantage of numerical methods is that their complexity grows with the dimension of the underlying system. By contrast analytic/algebraic conditions of the kind described in this paper involve a search over a parameter space whose size does not grow with the system dimension, and thus can provide a distinct advantage over numerical methods for high dimensional systems.

1.1 Definition of the problem

We restrict attention to one particular class of CQLF problems. Let $A \in \mathbb{R}^{n\times n}$ be a Hurwitz matrix, and $\{b_1,\dots,b_p,c_1,\dots,c_p \in \mathbb{R}^n\}$ a collection of vectors. Define

$$\mathcal{A} = \left\{ A - \sum_{i=1}^{p} x_i\, b_i c_i^T \,:\, x_i \geq 0 \right\} \tag{4}$$

(Comparing with (1), the index set $\mathcal{I}$ in (4) consists of p copies of the interval $[0,\infty)$.)

We consider the problem of finding necessary and sufficient conditions for the existence of a positive definite matrix $P = P^T > 0$ such that

$$P\left(A - \textstyle\sum_{i=1}^p x_i b_i c_i^T\right) + \left(A - \textstyle\sum_{i=1}^p x_i b_i c_i^T\right)^T P < 0 \quad \text{for all } x_1,\dots,x_p \geq 0 \tag{5}$$

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