Journal of Nonlinear Systems and Applications (2011) 35–43

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AN OPERATOR APPROACH TO VIABLE ATTAINABILITY OF HYBRID SYSTEMS

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Abstract. In this paper, the controllability operator approach to viability of hybrid systems is extended to consider the problems of attainability and viable attainability. In each case, a relation is defined that captures the hybrid system's behaviour over sampling intervals. Corresponding operators, the attainability operator and the time-independent viable attainability operator, are defined over the entire time interval of existence. This provides a unified approach to three constraint problems of hybrid systems.The development for attainability and viable attainability is examined through a three fluid filled tank example.

Keywords. Attainability, Viability, Hybrid Systems, Controllability Operator.

1 Introduction and Background

In [13], a *hybrid system* is taken to mean a system of continuous plants, subject to disturbances, interacting with sequential automaton. *Hybrid Control* is taken to mean control of continuous plants by control programs implemented on a sequential automaton. Of interest in this work is the design of sequential control automaton which ensure that two performance criteria are satisfied, these being viability and attainability.

The concept of viability is akin to controlled invariance in control systems. Viability is loosely defined to describe the ability of a control system to evolve in the interior of a user-defined subset, the viable set, of the state-space given an initial state in this subset. Attainability is the property of a control system that describes the ability to reach a user-defined subset, the target (or attainable) set, starting from an initial state lying outside this set. In this paper, we are interested in the design of control systems that achieve these two performance criteria simultaneously, i.e., the system state remains within some subset while acquiring the target set. We refer to this property as viable-attainability. A schematic representation of the viable-attainability problem is depicted in Figure 1.

The control mechanism employed to satisfy these two performance criteria is a sequential control automaton. The action of the control automaton is such that, at sampling times, the state variables of the system are measured and, based on the measurements, a control action from a finite set of possible control actions is initiated by the control automaton. This control vector remains active for the sampling period and the process repeats at that time.

It has been shown in [14],[10],[8],[11] that the problem of ensuring viability can be solved by demonstrating the existence of a fixed point of the controllability operator. This condition guarantees that there exists a finite control automaton that ensures viability.

The problem of achieving attainability for hybrid systems can be derived from the continuous–time result on attainability due to [7] (given in Theorem 3.1 below).

In treating hybrid systems, the continuous–time result must be modified to account for the following properties:

1. there are a finite set of possible control vectors.
2. measurements are available only at sample times.
3. the measured state is known to only within some e-ball of the true measurement, i.e., measurement error of magnitude e is assumed.
4. control vectors can be selected only at sample times.
5. the state space is partitioned into subsets which require identical control action.

In addition,it is assumed that the control process proceeds by taking a measurement at sampling times, deciding on a control vector that satisfies the viable attainability condition and applying this vector to the continuous–time plant. The process repeats at each sampling instant whereby only the current control action is applied to the plant before repeating the computation of generating the control action at the next sampling interval.

The problem of attainability has been treated in the literature as a target problem in [15],[16]. Viable attainability has been considered in the literature as characterization of what is referred to as the capture basin. This characterization has been performed for differential inclusions in [3],[1],[2] and for impulse control systems in [17],[4].

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†Manuscript received October 31, 2010; revised February 25, 2011.