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ROBUST STABILITY AND STABILIZATION OF A CLASS OF NONLINEAR DISCRETE-TIME STOCHASTIC SYSTEMS WITH STATE AND CONTROLLER DEPENDENT NOISE

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Abstract. A problem of robust state feedback stability and stabilization of nonlinear discrete-time stochastic processes is considered. The linear rate vector of a discrete-time system is perturbed by a nonlinear function that satisfies a quadratic constraint. Our objective is to show how linear constant feedback laws can be formulated to stabilize this type of nonlinear discrete-time systems and, at the same time maximize the bounds on this nonlinear perturbing function which the system can tolerate without becoming unstable.The state dependent diffusion is modeled by a normal sequence of identically independently distributed random variables. The new formulation provides a suitable setting for robust stabilization of nonlinear discrete-time systems where the underlying deterministic system satisfy the generalized matching conditions. Our method which is based on linear matrix inequalities (LMI's) is distinctive from the existing robust control and absolute stability techniques. Examples are given to demonstrate the results.

Keywords. Stability, Stabilization, Non-linear Systems, Stochastic Systems, Robust Control

1 Introduction

The stability analysis of both continuous and discrete-time stochastic systems has attracted many researchers in system science area for decades. Stochastic systems arise in a wide area of applications in control engineering such as filtering, adaptive systems and identification, and learning and image recognition etc. (see, for example [1, 2, 3] and the references therein for more details). Problems of this nature in the above mentioned research areas have been extensively studied (see for example, feedback stabilization for discrete-time nonlinear systems [4, 5, 6, 7], feedback stabilization of continuous systems [8, 9], robust H_∞ control [10, 11], robustness of exponential stability [12], optimal stabilizing compensator [13], among many).

The problem of robust quadratic stabilization of systems under nonlinear perturbation was studied for continuous systems [14] and for discrete-systems [15]. The solutions provided are for quadratically bounded nonlinear perturbations and are available for only deterministic systems. Even though a larger foundation has been laid out for stability and stabilization of discrete-time stochastic systems, thus far the problem of robust quadratic stabilization of discrete-time stochastic systems under nonlinear perturbation has been given very little attention.

The framework for robust quadratic stabilization of systems under nonlinear perturbation is closely related to absolute stability results in the context of Popov and Yakubovich [16, 17]. There is extensive literature and many techniques are available for the quadratic stabilization problems [18, 19]. However, the unknown structural dependency of the nonlinear perturbation on the state of the system prevented us from using some of these available tools of absolute stability in solving the robust stability problems under this new setting.

Unlike robust control results available in the literature [20, 21]. the result presented in this paper designs a linear control law that stabilizes the closed-loop system and is maximally robust with respect to nonlinear perturbations considered. Neither does the solution assume the linear component of the system to be stable, nor do the perturbations satisfy the matching conditions [14, 15]. Our results are also different from the literature available on robust control [20, 21]. Because of the fact that we do want to stabilize the system using a linear control law at the same time maximizes the class of uncertain perturbations which can be tolerated by the stabilized closed-loop system. We do not assume that the linear part is stable, nor that the perturbations satisfy the matching conditions [14, 15]. These assumptions allowed us to investigate alternative techniques compared to the existing well known robust control techniques.

The objective of this paper is to show how a solution to such complex control problem can be solved by the efficient tools of S-procedure [22] and the LMI techniques [23, 24]. Recent advances in the theory of linear matrix inequalities (LMI) has allowed numerous investigations in revisiting robust stability and stabilization of stochastic systems [24, 25, 26]. Although these results are

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