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A REMARK FOR DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. We give a proposal to generalize the concept of the differential equations on time scales, such that they can be more appropriate for the analysis of real world problems, and give more opportunities to increase the theoretical depth of investigation.

Keywords. time scale, variable time scale, transition condition, modeling

1 Differential equations on time scales in Hilger's sense and the first remark

Let us remind the differential equations on time scales, proposed by Hilger (see [11, 4, 12]). The main element of the equations is the time scale, which is understood as a nonempty closed subset, $\mathbb{T}$, of the real numbers. On a time scale $\mathbb{T}$, the functions $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$ and $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$ are called the forward and backward jump operators, respectively. The point $t \in \mathbb{T}$ is called right-scattered if $\sigma(t) > t$, and right-dense if $\sigma(t) = t$. Similarly, it is called left-scattered if $\rho(t) < t$, and left-dense if $\rho(t) = t$.

The $\Delta-$derivative of a continuous function φ, at a right-scattered point is defined as

$$\varphi^{\Delta}(t) := \frac{\varphi(\sigma(t)) - \varphi(t)}{\sigma(t) - t}, \qquad (1)$$

and at a right-dense point it is defined as

$$\varphi^{\Delta}(t) := \lim_{s \to t} \frac{\varphi(t) - \varphi(s)}{t - s}, \qquad (2)$$

if the limit exists.

A differential equation

$$y^{\Delta}(t) = f(t, y), \quad t \in \mathbb{T} \qquad (3)$$

is said to be a differential equation on time scale, where function $f(t, y) : \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ in (3) is assumed to be rd-continuous on $\mathbb{T} \times \mathbb{R}^n$ (see [7]).

If a point t is right-dense, then the Δ-derivative is the ordinary derivative. Otherwise, $t \in \mathbb{T}$ is a right-scattered point, and the Δ-derivative is defined by means of the quotient. Let us discuss the second case more attentively. By the definition of the $\Delta-$derivative, assuming that t is not an isolated point, one has that $\varphi(\sigma(t)) = \varphi(t) + \varphi'(t)(\sigma(t) - t)$. Now, compare this expression with the numerical approximation of the function on the interval $[t, \sigma(t)]$. Setting $\Delta t = \sigma(t) - t$ and $\Delta\varphi = \varphi(\sigma(t)) - \varphi(t)$, we have that $\Delta\varphi = \varphi'(t)\Delta t$. Thus, one can see that the idea of the $\Delta-$derivative, as well as of the $\nabla-$derivative is close to the basic one for the numerical analysis (see [16]). Figures 1 and 2 give the illustration.

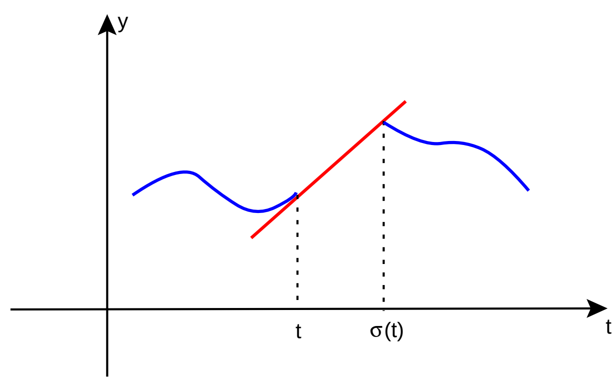

Figure 1:

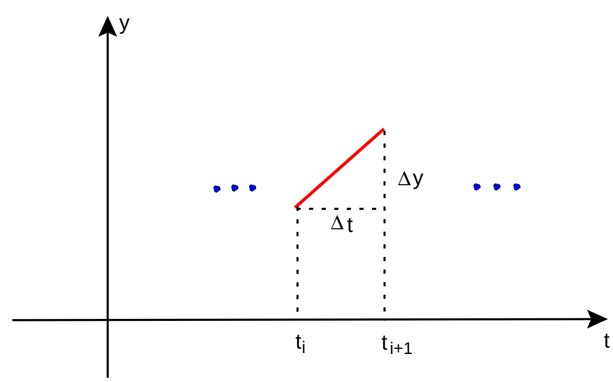

Figure 2:

That is, in its heart the concept of the differential equations on time scales is closely adjoint with numerical methods. However, for numerical methods, of course as far as the definition in (1) is considered, smaller $\sigma(t)-t$

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