

# DELAY-DEPENDENT ROBUST $H_\infty$ CONTROL OF NONLINEAR UNCERTAIN SYSTEMS WITH STATE AND INPUT DELAYS

Miaoxin Wang, Xinzhi Liu, Jizhen Liu, and Xiangjie Liu \*†‡§

**Abstract.** Delay-dependent robust  $H_\infty$  controller design problem for time-delayed systems with a class of nonlinear uncertainties is discussed in the paper. Assuming that the nonlinear uncertain functions in the considered model are gain-bounded, we obtain a sufficient condition for the robustly asymptotic stability and  $H_\infty$  performance of the closed-loop system in the light of Lyapunov stability theory. By solving a linear matrix inequality (LMI), we construct delay-dependent state feedback control law which guarantees the robustly asymptotic stability of the closed-loop system and reduce the effect of the disturbance input on the controlled output to a prescribed level. The delay-bound or the  $H_\infty$  performance bound can be optimized. A numerical example illustrates the potentiality of our claim.

**Keywords.** nonlinear systems, state and input delay, delay-dependent, robust  $H_\infty$  control, LMI

## 1 Introduction

It is well-known that time-delay is often encountered in various control systems, such as chemical processes, long transition lines in pneumatics, and rolling mill systems. Time-delay is usually results in unsatisfactory performances and is frequently a source of instability. Time-delay systems are also called systems with aftereffect or dead-time, hereditary systems, equations with deviating argument or difference equations (see [1]). Since the state space  $H_\infty$  controller design method based on linear matrix inequality (LMI) approach were proposed, many researchers have focused their interests on the problem of  $H_\infty$  controller design for time-delay systems (see [2]) and robust  $H_\infty$  controller design for time-delay systems with uncertainties (see [3]).

Depending on the information on the delay size in the property, one has two different kinds of asymptotic

stability for time-delay systems, i.e. delay-independent stability conditions (see [4]) versus delay-dependent ones (see [5-16]). Compared with delay-independent conditions, delay-dependent ones have less conservatism when the actual delay size is small. Ref. [6] provides a LMI approach to delay-dependent observer-based stabilizing and  $H_\infty$  controller design for linear state-delayed systems. The resulted delay-dependent criteria for the existence of controllers are presented by two independent LMIs via constructing a proper Lyapunov functional. Ref. [7] gives a new delay-dependent stability sufficient condition by using the Lyapunov-Krasovskii function method and free weighing matrix lines. Some free matrices are introduced in the derivative of the Lyapunov-Krasovskii function. Using the past state or past input information, Ref. [8] gives memory delay-dependent controllers, which are shown by examples to be less conservative than the memoryless controllers. Ref. [9] gives a sufficient condition in terms of linear matrix inequality for linear uncertainty time-varying delayed systems. The results dependent on the size of time-delay and on the size of time-derivative. Ref. [10] gives delay-dependent stability analysis for linear time delay system by using the Lyapunov-Krasovskii theory and LMI in both constant delay and time-varying delay cases.

The models considered in all of the above-mentioned works are linear ones, including the uncertainty parts of the models. The actual control systems have nonlinear characteristic besides time-delay effect. It is only for analysis and synthesis convenience that we use the linear models. Thus, it is more practical that the uncertain parts of a linear model should be described as nonlinear functions. But the problem of delay-dependent robust  $H_\infty$  control for nonlinear uncertainty systems has received little attention in the literature. Ref. [11] designs a delay-dependent robust  $H_\infty$  state feedback controller for a class of state nonlinear uncertain time-delay systems by using linear matrix inequality (LMI) based on an appropriate Lyapunov function and Lyapunov stability theory. Ref. [12] designs full and reduced delay-dependent robust  $H_\infty$  filter for nonlinear uncertainty time delay Takagi-Sugeno fuzzy models. Sufficient conditions for the existence of a delay-dependent  $H_\infty$  filter are given in terms of linear matrix inequalities. The models considered in the two works

\*Miaoxin Wang is with School of Control and Computer Engineering, North China Electric Power University, Beijing, China. He currently is a Post-doctoral Fellow with Department of Applied Mathematics, University of Waterloo, Canada. E-mails: m68wang@uwaterloo.ca

†Xinzhi Liu is with Department of Applied Mathematics, University of Waterloo, Canada. E-mail: xzliu@uwaterloo.ca

‡Jizhen Liu and Xiangjie Liu are with School of Control and Computer Engineering, North China Electric Power University, Beijing, China. E-mails: jzliu@ncepu.edu.cn, liuxj@ncepu.edu.cn

§Manuscript received April 19, 2010; revised February 28, 2011.