ON A METHOD OF RESOLUTION OF CONTROLLABILITY PROBLEMS FOR A POPULATION DYNAMICS EQUATION

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Abstract. We are interested in the control of the population dynamics problem in order to subject its solution to constraints, here a finite number of linear constraints. It is a problem of the type controllability that we analyze in two steps. First, we interpret each linear constraint by the adjoint state. Then we show that the problem of control on the state is equivalent to a problem of constraint on control that we solve.

Keywords. Population dynamics, controllability, sentinels, Carleman’s inequality

1 Introduction

Let $N, M \in \mathbb{N}^*$ and $\Omega$ be a bounded open subset of $\mathbb{R}^N$ with boundary $\Gamma = \partial \Omega$ of class $C^2$. Let $\omega \subset \Omega$ be an open non empty subset. For a given positive real function $F$ and for a time $T > 0$, we consider the following population dynamics problem:

$$
\begin{align*}
\frac{\partial y}{\partial t} + \frac{\partial y}{\partial a} - \Delta y + \mu y &= v \chi_\omega \text{ in } (0, T) \times (0, A) \times \Omega, \\
y(0, a, x) &= y_0(a, x) \text{ in } (0, A) \times \Gamma, \\
y(t, 0, x) &= \int_0^A \beta(t, a, x) y(t, a, x) da \text{ in } (0, T) \times \Omega.
\end{align*}
$$

(1)

The number $y(t, a, x)$ is the distribution of $a - \text{year}$ old individuals at time $t$ at the point $x$, $A$ is the maximal life expectancy. The weight $\beta = \beta(t, a, x)$ is the natural fertility rate, the function $\mu = \mu(t, a, x)$ is the natural death rate of $a - \text{year}$ old individuals at time $t > 0$ in

the position $x$. Thus, the formula $\int_0^A \beta(t, a, x) y(t, a, x) da$

denotes the distribution of newborn at time $t$ and position $x$. We set $U = (0, T) \times (0, A)$, $Q = U \times \Omega$, $Q_A = (0, A) \times \Omega$, $Q_T = (0, T) \times \Omega$, $\Sigma = U \times \Gamma$, $\mathcal{G} = U \times \omega$, the function $y^0 \in L^2(Q_A)$, the control $v$ belongs to $L^2(Q)$ and $\chi_\omega$ is the characteristic function of the control set $\omega$.

For the sequel we assume that the following assumptions hold:

$$
\begin{align*}
\mu(t, a, x) &= \mu_0(a) + \mu_1(t, a, x) \text{ in } U \times \Omega, \\
\mu &\geq 0 \text{ in } U \times \Omega, \\
\mu_1 &\in L^\infty(U \times \Omega); \mu_1(t, a, x) \geq 0 \text{ in } U \times \Omega, \\
\mu_0 &\in L^1_{\text{loc}}(0, A), \lim_{a \to A} \int_0^a \mu_0(s) ds = +\infty
\end{align*}
$$

(2)

$$
\begin{align*}
\beta &\in C^2([0, T] \times [0, A] \times \overline{\Omega}), \\
\beta(t, a, x) &\geq 0 \text{ in } [0, T] \times [0, A] \times \overline{\Omega}, \\
\exists 0 < a_0 < a_1 < A \text{ such that } \\
\beta(t, a, x) &= 0 \text{ in } [0, T] \times ([0, a_0) \cup (a_1, A) \times \Omega).
\end{align*}
$$

(3)

Since $\mu$ and $\beta$ are natural rates, the second assumptions of $H_1$ and $H_2$ are natural. The third assumption of $H_2$ is also natural, since it means that older and younger individuals are not fertile. The fourth assumption in $H_1$ is also a standard one, it means that all individual dies before the age $A$.

The null controllability problem can be formulated as follows: Given $y^0 \in L^2(Q_A)$, find $v \in L^2(Q)$ such that the solution of (1) satisfies

$$y(T, a, x) = 0 \text{ in } (0, A) \times \Omega$$

This kind of controllability problem has been widely studied. Let us mention briefly some of the existing works. The first controllability result for an age and structured population dynamics model was established by Ainseba and Langlais [3]. They proved that a set of profiles is approximately reachable. In [5] a local exact controllability result is obtained. Actually, they authors show...