Journal of Nonlinear Systems and Applications (2011) 205–209
 http://www.watsci.org/JNSA/

ON A METHOD OF RESOLUTION OF CONTROLLABILITY PROBLEMS FOR A POPULATION DYNAMICS EQUATION

Gisèle. M. Mophou and Sawadogo Somdouda *†‡

Abstract. We are interested in the control of the population dynamics problem in order to subject its solution to constraints, here a finite number of linear constraints. It is a problem of the type controllability that we analyze in two steps. First, we interpret each linear constraint by the adjoint state. Then we show that the problem of control on the state is equivalent to a problem of constraint on control that we solve.

Keywords. Population dynamics, controllability, sentinels, Carleman's inequality

1 Introduction

Let N, $M \in \mathbb{N}^\star$ and Ω be a bounded open subset of $\mathbb{R}^N$ with boundary $\Gamma = \partial\Omega$ of class $\mathcal{C}^2$. Let $\omega \subset \Omega$ be an open non empty subset. For a given positive real function F and for a time $T > 0$, we consider the following population dynamics problem:

$$\begin{cases} \dfrac{\partial y}{\partial t} + \dfrac{\partial y}{\partial a} - \Delta y + \mu y = v\chi_\omega \text{ in } (0,T)\times(0,A)\times\Omega, \\ y = 0 \text{ on } (0,T)\times(0,A)\times\Gamma, \\ y(0,a,x) = y^0(a,x) \text{ in } (0,A)\times\Omega, \\ y(t,0,x) = \int_0^A \beta(t,a,x)y(t,a,x)da \text{ in } (0,T)\times\Omega. \end{cases} \tag{1}$$

The number $y(t,a,x)$ is the distribution of $a - year$ old individuals at time t at the point x, A is the maximal life expectancy. The weight $\beta = \beta(t,a,x)$ is the natural fertility rate, the function $\mu = \mu(t,a,x)$ is the natural death rate of $a-year$ old individuals at time $t > 0$ and in the position x. Thus, the formula $\int_0^A \beta(t,a,x)y(t,a,x)da$ denotes the distribution of newborn at time t and position x. We set $U = (0,T)\times(0,A)$, $Q = U\times\Omega$, $Q_A = (0,A)\times\Omega$, $Q_T = (0,T)\times\Omega$, $\Sigma = U\times\Gamma$, $\mathcal{G} = U\times\omega$. the function $y^0 \in L^2(Q_A)$, the control v belongs to $L^2(Q)$ and χ_ω is the characteristic function of the control set ω.

For the sequel we assume that the following assumptions hold:

$$\begin{cases} \mu(t,a,x) = \mu_0(a) + \mu_1(t,a,x) \text{ in } U\times\Omega, \\ \mu \geq 0 \text{ in } U\times\Omega, \\ \mu_1 \in L^\infty(U\times\Omega);\ \mu_1(t,a,x) \geq 0 \text{ in } U\times\Omega, \\ \mu_0 \in L^1_{loc}(0,A),\ \lim\limits_{a\to A}\int_0^a \mu_0(s)ds = +\infty \end{cases} \tag{2}$$

$$\begin{cases} \beta \in C^2([0,T]\times[0,A]\times\overline{\Omega}), \\ \beta(t,a,x) \geq 0 \text{ in } [0,T]\times[0,A]\times\overline{\Omega}, \\ \exists\, 0 < a_0 < a_1 < A \text{ such that} \\ \beta(t,a,x) = 0 \text{ in } [0,T]\times((0,a_0)\cup(a_1,A)\times\Omega). \end{cases} \tag{3}$$

Since μ and β are natural rates, the second assumptions of H_1 and H_2 are natural. The third assumption of H_2 is also natural, since it means that older and younger individuals are not fertile. The fourth assumption in H_1 is also a standard one, it means that all individual dies before the age A.

The null controllability problem can be formulated as follows: *Given $y^0 \in L^2(Q_A)$, find $v \in L^2(Q)$ such that the solution of (1) satisfies*

$$y(T,a,x) = 0 \text{ in } (0,A)\times\Omega$$

This kind of controllability problem has been widely studied. Let us mention briefly some of the existing works. The first controllability result for an age and structured population dynamics model was established by Ainseba and Langlais [3]. They proved that a set of profiles is approximately reachable. In [5] a local exact controllability result is obatained. Actually, they authors show

*Gisèle M. Mophou is with Laboratoire C.E.R.E.G.M.I.A., Université des Antilles et de la Guyane, Campus Fouillole 97159 Pointe-à-Pitre Guadeloupe (FWI). E-mail: gmophou@univ-ag.fr

†Sawadogo Somdouda is with Département de Mathématiques et Informatique, UFR/Sciences Exactes et Appliquées, Université de Ouagadougou.Adresse personelle: 09 BP 522 Ouaga 09-Burkina Faso. E-mail: sawasom@yahoo.fr

‡Manuscript received January 5, 2011; accepted April 16, 2011.