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PSEUDO ALMOST AUTOMORPHIC SOLUTIONS FOR DIFFERENTIAL EQUATION WITH PIECEWISE CONSTANT ARGUMENT IN A BANACH SPACE

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Abstract. We give sufficient conditions for the pseudo almost automorphy of bounded solutions to differential equations with piecewise constant argument of the form $x'(t) = A(t)x([t]) + f(t), t \in \mathbb{R}$, where A(t) is a pseudo almost automorphic operator, f(t) is a $\mathbb{X}$-valued pseudo almost automorphic function and $\mathbb{X}$ is a finite dimensional Banach space. We give also sufficient conditions for the uniform stability of this equation.
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1 Introduction

The study of differential equations with piecewise constant arguments (EPCA) is an important subject because these equations have the structure of continuous dynamical systems in intervals of unit length. Therefore they combine the properties of both differential equations and difference equations. There have been many papers studying EPCA. The first work devoted to this subject is the paper of Shah and Wiener [13] in 1983. In 1984 Cook and Wiener in [3] study these equations with delay. In [15] Nguyen Van minh and Tran Tat Dat give sufficient spectral conditions for the almost automorphy of bounded solutions of the equations of the form

$$x'(t) = Ax([t]) + f(t)$$

where A is a bounded linear operator in a finite dimensional Banach space $\mathbb{X}$, f is an $\mathbb{X}$-valued almost automorphic function and [.] is the largest integer function. In [4], we generalize [15] giving sufficient spectral conditions for the almost automorphy of bounded solutions of the equations of the form

$$x'(t) = A(t)x([t]) + f(t) \qquad (1)$$

where $f(t)$ is an $\mathbb{X}$-valued almost automorphic function and $A(t)$ is an $\mathbb{X}$-valued almost automorphic operator. In this paper we give sufficient conditions for the pseudo almost automorphy of bounded solution of (1). Therefore we generalize some results of [4] because an almost automorphic function is a pseudo almost automorphic function.

The rest of this paper is organized as follows. First of all, we will recall the concept of almost automorphic functions and almost automorphic sequences. We give results obtained in [4]. Section 3 presents the main results of this paper. Theorem 2 presents sufficient condition in order to obtain pseudo almost automorphy solution. Theorem 4 gives sufficients conditions for the uniform stability of (1).

2 Preliminaries

2.1 Almost automorphic functions

Definition 1. *(S.Bochner) A continuous function $f : \mathbb{R} \to \mathbb{X}$ is said to be almost automorphic if for any sequence of real numbers (s'_n), there exists a subsequence (s_n) such that*

$$\lim_{m\to\infty} \lim_{n\to\infty} f(t + s_n - s_m) = f(t) \qquad (2)$$

for any $t \in \mathbb{R}$.
The limit in (2) means that

$$g(t) = \lim_{n\to\infty} f(t + s_n) \qquad (3)$$

is well defined for each $t \in \mathbb{R}$ and

$$f(t) = \lim_{n\to\infty} g(t - s_n)$$

for each $t \in \mathbb{R}$.
If the limit is uniform on any compact subset $K \subset \mathbb{R}$, we say that f is compact almost automorphic. We denote by $AA(\mathbb{X})$ the space of all continuous almost automorphic functions $f : \mathbb{R} \to \mathbb{X}$.

Remark 1. *If we equip $AA(\mathbb{X})$ with the sup-norm*

$$||f||_\infty = \sup_{t\in\mathbb{R}} ||f(t)||$$

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