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OPTIMAL CONTROL OF GENERALIZED BOLZA PROBLEM FOR SEMILINEAR EVOLUTION INCLUSIONS

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Abstract. This paper studies a general optimal control problem of Bolza for semilinear evolution inclusions with initial condition and endpoint constraint in reflexive and separable Banach spaces. We employ the method of discrete approximations and advanced tools of generalized differentiation in infinite-dimensional spaces to derive necessary optimality conditions in the extended Euler-Lagrange form.

Keywords. optimal control, variational analysis, generalized differentiation, semilinear evolution inclusions, discrete approximations, necessary optimality conditions.

1 Introduction

Let X be a reflexive and separable Banach space, and let $F\colon X \times [a,b] \rightrightarrows X$ be a set-valued mapping. The primary object of this paper is to study the following *generalized Bolza problem* (P) for *semilinear evolution inclusions* with general endpoint constraints:

$$\text{minimize} \quad J[x] := \varphi(x(b)) + \int_a^b f(x(t),t)dt \qquad (1)$$

over *mild* continuous trajectories $x\colon [a,b] \to X$ for the semilinear evolution inclusion

$$\dot{x}(t) \in Ax(t) + F(x(t),t), \quad x(a) = x_0 \in X \qquad (2)$$

subject to the endpoint constraints

$$x(b) \in \Omega \subset X, \qquad (3)$$

where $A\colon X \to X$ is an *unbounded* generator of the C_0-semigroup $\{e^{At} | \ t \geq 0\}$. In particular, when $F(x,t) = f(x,U,t)$ with the set U a control, this relates (2) to semilinear control evolution *equations* considered in PDE control theory for *smooth* data; see, e.g., the books by Fattorini [3], by Li and Yong [6], for comprehensive discussions and additional references therein.

The primary goal of this paper is to employ the *method of discrete approximations* developed by Mordukhovich [7] to study a Bolza type optimal control problem in infinite dimensional spaces. Later this method was extended in [9, 10, 15, 16, 17, 18] to more general situations In particular, in Mordukhovich and Wang [15, 16], this method was extended to studying optimal control problems of minimizing a Mayer-type function over appropriate solutions to the semilinear evolution inclusion (2) with endpoint constraint (3). The *unboundedness* of the operator A in (2) is crucial for applications to control problems for partial differential equations and inclusions.

The rest of the paper is organized as follows. In Section 2, using the result of strong approximation by discrete trajectories developed in [7] and later extended to optimal control systems governed by the semilinear evolution inclusions in [15, 16], for the given reference optimal solution $\bar{x}(\cdot)$ to (P) we construct a well-posed sequence of discrete approximations $\{(P_N)\}, N = 1, 2, \ldots,$ to (P) in such a way that each (P_N) admits an optimal solution $\bar{x}_N(\cdot)$ and the sequence $\{\bar{x}_N(\cdot)\}$ converges to $\bar{x}(\cdot)$ strongly in $C([a,b];X)$ as $N \to \infty$.

In Section 3 we briefly review basic tools of generalized differentiation in variational analysis needed for deriving necessary optimality conditions in discrete approximation problems and then establishing, by passing to the limit, adequate necessary conditions for optimality of the given solution to (P).

Section 4 is devoted to necessary optimality conditions for the discrete-time problems appeared in our discrete approximation procedure. We pay the main attention to "fuzzy" (or approximate) optimality conditions in discrete approximations that are more flexible and require less assumptions for the subsequent derivation of optimality conditions for evolution inclusions by passing to the limit.

In Section 5 we develop the limiting procedure and establish necessary optimality conditions for the original continuous-time problem (P) by passing to the limit from discrete approximations. In this way we obtain new conditions in the *extended Euler-Lagrange form* involving mild solutions to a certain *adjoint* evolution inclusion.

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