OPTIMAL CONTROL OF GENERALIZED BOLZA PROBLEM FOR SEMILINEAR EVOLUTION INCLUSIONS

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Abstract. This paper studies a general optimal control problem of Bolza for semilinear evolution inclusions with initial condition and endpoint constraint in reflexive and separable Banach spaces. We employ the method of discrete approximations and advanced tools of generalized differentiation in infinite-dimensional spaces to derive necessary optimality conditions in the extended Euler-Lagrange form.

Keywords. optimal control, variational analysis, generalized differentiation, semilinear evolution inclusions, discrete approximations, necessary optimality conditions.

1 Introduction

Let $X$ be a reflexive and separable Banach space, and let $F : X \times [a, b] \to X$ be a set-valued mapping. The primary object of this paper is to study the following generalized Bolza problem $(P)$ for semilinear evolution inclusions with general endpoint constraints:

$$
\text{minimize } J[x] := \varphi(x(b)) + \int_a^b f(x(t), t)dt \quad (1)
$$

over mild continuous trajectories $x : [a, b] \to X$ for the semilinear evolution inclusion

$$
\dot{x}(t) \in Ax(t) + F(x(t), t), \quad x(a) = x_0 \in X \quad (2)
$$

subject to the endpoint constraints

$$
x(b) \in \Omega \subset X, \quad (3)
$$

where $A : X \to X$ is an unbounded generator of the $C_0$-semigroup $\{e^{tA} | t \geq 0\}$. In particular, when $F(x, t) = f(x, U, t)$ with the set $U$ a control, this relates (2) to semilinear control evolution equations considered in PDE control theory for smooth data; see, e.g., the books by Fattorini [3], by Li and Yong [6], for comprehensive discussions and additional references therein.

The primary goal of this paper is to employ the method of discrete approximations developed by Mordukhovich [7] to study a Bolza type optimal control problem in infinite dimensional spaces. Later this method was extended in [9, 10, 15, 16, 17, 18] to more general situations.

In Section 1 we develop the limiting procedure and establish necessary optimality conditions for the original continuous-time problem $(P)$ by passing to the limit from discrete approximations. In this way we obtain new conditions in the extended Euler-Lagrange form involving mild solutions to a certain adjoint evolution inclusion.