Journal of Nonlinear Systems and Applications (2011) 223–228

http://www.watsci.org/JNSA/

STABILITY OF SOLUTIONS OF NEUTRAL SPDEs

T.E. Govindan *†

Abstract. In this paper, we study neutral stochastic partial differential equations (SPDEs). Assuming the existence and uniqueness of a mild solution, our aim here is to investigate the almost sure exponential stability of mild solutions of SPDEs by using the Ousgood and linear growth conditions on the nonlinear terms.
Keywords. Neutral stochastic partial differential equations, mild solution, linear growth condition, Ousgood type condition, almost sure exponential stability.

1 Introduction

In this paper, we study a neutral stochastic partial differential equation in a real Hilbert space of the form

$$d[x(t) + f(t, x_t)] = [Ax(t) + a(t, x_t)]dt + b(t, x_t)dw(t), \quad t > 0; \quad (1)$$

$$x(t) = \varphi(t), \quad t \in [-r, 0] \quad (0 \le r < \infty); \quad (2)$$

where $x_t(s) = x(t+s)$, $-r \le s \le 0$, $-A : D(-A) \subset X$ is the infinitesimal generator of a strongly continuous semigroup $\{S(t), t \ge 0\}$ defined on $X, a : R^+ \times C \to X$ $(R^+ = [0, \infty))$, $f : R^+ \times C \to D((-A)^{-\alpha})$, $0 < \alpha \le 1$, see Pazy [10, pp. 69-75] for definition and properties and $b : R^+ \times C \to L(Y, X)$ are Borel measurable; and for each (t, u) are measurable with respect to the σ-algebra B_t. Here $w(t)$ is a Y-valued Q-Wiener process and the past stochastic process $\{\varphi(t), t \in [-r, 0]\}$ has almost sure (a.s.) continuous paths with $E||\varphi||_C^2 < \infty$.

Equation (1) was studied in Luo [9] and Govindan [3, 6] by using global Lipschitz conditions on all the nonlinear terms; and in Bao and Hou [1] and Govindan [4, 7] using non-Lipschitz conditions on $a(t, u)$ and $b(t, u)$. Note that when $f = 0$, equation (1) has been well studied, see for instance, Govindan [5] and the references therein.

The rest of the paper is organized as follows: In Section 2, we give the preliminaries following Taniguchi [12]. Since we work in the same framework as in Govindan [3, 4, 6, 7] we shall be quite brief here. In Section 3, we present our main result.

*The author is with the Department of Mathematics, School of Physics and Mathematics, National Polytechnic Institute, Mexico City, 07738, Mexico. tegovindan@yahoo.com
†Manuscript received January 4, 2011; accepted November 10, 2011.

2 Problem statement and preliminaries

In this article, our goal is to study the almost sure exponential stability of the problem (1)-(2) using linear growth and non-Lipschitz conditions on the drift and diffusion terms while on f we assume the Lipschitz condition.

Let X, Y be real separable Hilbert spaces and $L(Y, X)$ be the space of bounded linear operators mapping Y into X. We shall use the same notation $|\cdot|$ to denote norms in X, Y and $L(Y, X)$. Let $(\Omega, B, P, \{B_t\}_{t \ge 0})$ be a complete probability space with an increasing right continuous family $\{B_t\}_{t \ge 0}$ of complete sub-σ-algebras of B. Let $\beta_n(t)(n = 1, 2, 3, \ldots)$ be a sequence of real-valued standard Brownian motions mutually independent defined on this probability space. Set $w(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\beta_n(t)e_n$, $t \ge 0$, where $\lambda_n \ge 0$ $(n = 1, 2, 3, \ldots)$ are nonnegative real numbers and $\{e_n\}$ $(n = 1, 2, 3, \ldots)$ is a complete orthonormal basis in Y. Let $Q \in L(Y, Y)$ be an operator defined by $Qe_n = \lambda_n e_n$. The above $Y-$valued stochastic processes $w(t)$ is called a Q-Wiener process. Let $h(t)$ be an $L(Y, X)$–valued function and let λ be a sequence $\{\sqrt{\lambda_1}, \sqrt{\lambda_2}, \ldots\}$, Then we define $|h(t)|_\lambda = \left\{\sum_{n=1}^{\infty} |\sqrt{\lambda_n}h(t)e_n|^2\right\}^{1/2}$. If $|h(t)|_\lambda^2 < \infty$, then $h(t)$ is called λ-Hilbert-Schmidt operator.

A semigroup $\{S(t), t \ge 0\}$ is said to be exponentially stable if there exist positive constants M and a such that $||S(t)|| \le M\exp(-at)$, $t \ge 0$, where $||\cdot||$ denotes the operator norm in X. If $M = 1$, the semigroup is said to be a contraction.

Let $C := C([-r, 0]; X)$ denote the space of continuous functions $\varphi : [-r, 0] \to X$ endowed with the norm $||\varphi||_C = \sup_{-r \le s \le 0} |\varphi(s)|$.

Definition 2.1 *A stochastic process* $\{x(t), t \in [0, T]\}(0 < T < \infty)$ *is called a mild solution of equation (1) if* i) $x(t)$ is B_t-adapted with $\int_0^T |x(t)|^2 dt < \infty$, a.s., and ii) $x(t)$ satisfies the integral equation

$$x(t) = S(t)[\varphi(0) + f(0, \varphi)] - f(t, x_t)$$