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STANDING WAVE SOLUTIONS OF THE DISCRETE NONLINEAR SCHRÖDINGER EQUATIONS WITH SIGN CHANGING HOMOGENEOUS NONLINEARITY

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Abstract. In this paper we investigate the standing wave solutions of the discrete nonlinear Schrödinger equation with the unbounded potential and homogeneous nonlinearity containing both self-focusing and defocusing sites (which means the coefficients of nonlinearity change sign). By using the generalized Nehari manifold method we obtain the existence of nontrivial exponential decay standing wave solutions.

Keywords. Standing wave, discrete nonlinear Schrödinger equation, sign changing homogeneous nonlinearity, unbounded potential, generalized Nehari manifold.

1 Introduction

1.1 Physical background

In [27, 28, 29] we investigated the standing wave solutions of the discrete nonlinear Schrödinger equation with either self-focusing or defocusing nonlinearity. In this paper we consider the case of the homogeneous nonlinearity containing both self-focusing and defocusing sites which means the coefficients of nonlinearity change sign and is named sign changing nonlinearity. For simplicity, we consider the one dimensional discrete nonlinear Schrödinger (DNLS) equation,

$$i\dot{\psi}_n + \Delta\psi_n - v_n\psi_n + \gamma_n f(\psi_n) = 0, \quad n \in \mathbb{Z},\ 2 < p < \infty \tag{1.1}$$

and

$$\Delta\psi_n = \psi_{n+1} - 2\psi_n + \psi_{n-1} \tag{1.2}$$

is the discrete Laplacian operator.

The DNLS equation (1.1) is one of the most important inherently discrete models, having a crucial role in the modelling of a great variety of phenomena, ranging from solid-state and condensed-matter physics to biology [2, 3, 4, 5, 6, 8, 9, 13, 21]. Some reviews on DNLS equatios can be found in [3, 12], especially both theoretical and numerical simulation results can be found in [12]. Recently Pankov [14, 15, 17] systematically studied both periodic and decaying solutions. The nonlinear Schrödinger equation with growing potential has been extensively investigated by many mathematicians and physicists, from the fundamental wellposedness of Cauchy problem [23, 24, 25] to the existence and stability of standing waves [7, 16, 18, 26].

1.2 Assumptions

Assume that the nonlinearity $f(u)$ is gauge invariant, that is, $f(e^{i\omega}u) = e^{i\omega}f(u)$ for any $\omega \in \mathbb{R}$. We consider the special solutions of (1.1) of the form $\psi_n = e^{-it\omega}u_n$. These solutions are called *standing waves* or breather solutions. Inserting the ansatz of standing waves into (1.1) we see that any standing wave solution satisfies the infinite nonlinear system of algebraic equations

$$-\Delta u_n + v_n u_n - \omega u_n - \gamma_n f(u_n) = 0 \tag{1.3}$$

(A1) Assume that the discrete potential $V = \{v_n\}_{n\in\mathbb{Z}}$ is bounded from below and satisfies

$$\lim_{|n|\to\infty} v_n = \infty. \tag{1.4}$$

(A2) Assume that $\gamma = \{\gamma_n\}_{n\in\mathbb{Z}} \in l^\infty(\mathbb{Z})$. Let

$$\Gamma_\pm = \{n \in \mathbb{Z} \mid \pm\gamma_n > 0\},\ \Gamma_0 = \{n \in \mathbb{Z} \mid \gamma_n = 0\}.$$

To study the standing wave solutions, we consider the real sequence spaces

$$\begin{aligned} l^p \equiv l^p(\mathbb{Z}) = \{u &= \{u_n\}_{n\in\mathbb{Z}} :\ \forall n \in \mathbb{Z}, u_n \in \mathbb{R}, \\ \|u\|_{l^p} &= (\sum_{n\in\mathbb{Z}} |u_n|^p)^{1/p} < \infty\}. \end{aligned}$$

Between l^p spaces the following elementary embedding relation holds

$$l^q \subset l^p, \quad \|u\|_{l^p} \le \|u\|_{l^q}, \quad 1 \le q \le p \le \infty.$$

Without losing the generality we assume that $V \ge 1$ and denote $H = -\Delta + V$ which is well-defined on $l^2(\mathbb{Z})$. Let

$$\begin{aligned} E &= \{u \in l^2(\mathbb{Z}) :\ (-\Delta + V)^{1/2}u \in l^2(\mathbb{Z})\}, \\ \|u\|_E &= \|(-\Delta + V)^{1/2}u\|_{l^2(\mathbb{Z})}. \end{aligned}$$

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