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# ANALYTICAL SOLUTIONS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS USING VARIATIONAL ITERATION METHOD

H. Jafari* and C. M. Khalique †‡

**Abstract.** Fractional calculus has been used to model physical and engineering processes that are found to be best described by fractional differential equations. For that reason we need a reliable and efficient technique for the solution of fractional differential equations. In this article, we implement the variational iteration method for solving fractional differential equations such as the Bagley-Torvik equation which arises in the modeling of the motion of a rigid plate immersed in a Newtonian fluid. The fractional derivatives are described in the Caputo sense.

**Keywords.** Caputo fractional derivative; Fractional differential equation; Variational iteration method; Bagley-Torvik equation, Fractional Riccati differential equation.

## 1 Introduction

Fractional differential equations, both partial and ordinary ones, have received much attention in recent years. Various phenomena in physics, like diffusion in a disordered or fractal medium have been modeled by means of fractional differential equations. In this connection, it is worthwhile to mention that the recent papers on numerical solutions of fractional differential equations are available from the notable works of Diethelm *et al.* [2, 3] and Jafari [5]. Recently, applications have included classes of nonlinear fractional differential equations [12] and their numerical solutions have been established by Diethelm and Ford [2, 3]. Analytical solution and also numerical solution of Bagley-Torvik equation were well established by Podlubny [12]. Jafari *et al.* [5] presented a numerical solution of non-linear Riccati differential equation. In the earlier work [2], Diethelm and Ford proposed a numerical solution of the Bagley-Torvik equation . Later they reviewed and generalized their previous work in the paper [3].

Although several authors proposed different techniques for the solution of Bagley-Torvik equation, yet in the present analysis an attempt has been made to obtain the solution by a different method.

Recently developed technique of variational iteration method [6–9] has proven to be a powerful method, and has successfully been applied in a variety of problems. The variational iteration method offers certain advantages over routine numerical methods. Numerical methods use discretization which gives rise to rounding off errors causing loss of accuracy, and requires large computer power and time. The variational iteration method is better since it does not involve discretization of the variables, hence is free from rounding off errors and does not require large computer memory or time. In this paper the variational iteration method has been employed to obtain solution of Bagley-Torvik equation [1, 12]

$$A(t)D_*^2 y(t) + B(t)D_*^{1.5} y(t) + C(t)y(t) = f(t). \qquad (1)$$

We consider the case $f(t) = C(t)(1+t), A(t) = 1, B(t) = 1$ and $C(t) = 1$ with initial conditions

$$y(0) = 1, \qquad y'(0) = 1. \qquad (2)$$

We also consider here the following non-linear fractional Riccati differential equation

$$D_*^\alpha y(t) = A(t) + B(t)y + C(t)y^2, \qquad (3)$$

subject to the initial conditions

$$y^{(k)}(0) = c_k, \quad k = 0, 1, \cdots, n-1, \qquad (4)$$

where $\alpha$ is fractional derivative order, $n$ is an integer, $A(t)$, $B(t)$ and $C(t)$ are known real functions, and $c_k$ are constants.

## 2 Basic Definitions

**Definition 2.1.** The left sided Riemann-Liouville fractional integral of order $\mu \geq 0$, [12] of a function $f \in C_\alpha, \alpha \geq -1$ is defined as:

$$I^\mu f(x) = \frac{1}{\Gamma(\mu)} \int_0^x \frac{f(t)}{(x-t)^{1-\mu}} dt, \ \mu > 0, \ x > 0, \qquad (5)$$
$$I^0 f(x) = f(x).$$

*H. Jafari is with Department of Applied Mathematics, University of Mazandaran, Iran. E-mails: jafari@umz.ac.ir, jafari_h@math.com

†C. M. Khalique is with International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735, Republic of South Africa. E-mail: Masood.Khalique@nwu.ac.za

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