Journal of Nonlinear Systems and Applications (2011) 160–166
 http://www.watsci.org/JNSA/

GLOBAL MATHEMATICAL ANALYSIS OF AN HIV INFECTION MODEL WITH FULL LOGISTIC GROWTH AND SATURATION INCIDENCE

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Abstract. In this research, we study an HIV infection mathematical model with a full logistic growth and a saturation incidence. Both local and global mathematical analysis are carried out. By identifying a basic reproduction number R_0, the existence and stability of the uninfected steady state P_0 and the unique infected steady state P^* are established in terms of R_0. We show that if $R_0 \leq 1$; P_0 is the only equilibrium in the feasible region, it is globally asymptotically stable. Therefore, no HIV infection persists and infected T cells and HIV virus are cleared over time. However, If $R_0 > 1$, then the unique infected steady state P^* emerges. P_0 becomes unstable. We show that the system is uniformly persistent and HIV infection persists. The unique infected steady state P^* is globally asymptotically stable under some conditions.
Keywords. HIV infection model, global analysis, steady states, uniform persistence, stability.

1 Introduction

The most notorious disease of AIDS/HIV has been studied intensively for the last several decades. Researchers are working diligently to find a cure for the disease from many different aspects including mathematical viewpoint. Different mathematical models that describe the interaction of HIV virus and T cells have been developed and studied, see for instance [1-11] and references therein. Typically, to describe the HIV infection, we partition the total T cell population into healthy (uninfected) T cells and infected T cells and study the dynamics between healthy T cells, infected T cells, and HIV free virus with their concentrations represented by $T(t), T^*(t)$ and $V(t)$ at time t, respectively. Then a basic mathematical model describing the HIV infection can be written in the following form,

$$\begin{aligned}\dot{T} &= s - \alpha T - kVT,\\ \dot{T}^* &= kVT - \beta T^*,\\ \dot{V} &= N\beta T^* - \epsilon V,\end{aligned} \tag{1}$$

where all parameters $s, \alpha, \beta, \epsilon, k, N$ are nonnegative. The human body produces T cells and it is assumed that T cells are produced at a constant rate s and that newly produced T cells are healthy and susceptible to HIV virus. Parameters α, β, and ϵ are the per-capita death rates of the healthy T cells, infected T cells, and the virus particles, respectively. It is natural to assume that $\alpha \leq \beta$, i.e. the infected T cells have a greater death rate than healthy cells because of the infectivity of the T cells. The infection is through a direct contact between virus and healthy T cells. The incidence is described by a simple mass-action term kVT, where $k > 0$ is the contact rate between virus and healthy T cells. Infected T cells produce virus and each infected T cell produces N virus during its lifetime.

It is well know now that T cells proliferate once stimulated by antigen or mitogen. Even though this proliferation process is still not well understood, a logistic proliferation form has been widely used in literature. Perelson and Nelson [3] and Leenheer and Smith [8] studied model (1) with a simplified logistic form, $rT\,(1 - T/T_{\max})$, where T_{max} is the capacity T cell population. With this simplified logistic form, the system is competitive and a complete mathematical analysis is carried out by Leenheer and Smith [8] using the theory for three dimensional competitive systems. Recently, Wang and Li [9] proposed and studied model (1) with a full logistic proliferation term, $rT\,(1 - (T + T^*)/T_{\max})$, and Wang and Ellermeyer [10] proposed and studied a similar model that utilized the full logistic form in both the first and second equations assuming that both the healthy and infected T cells proliferate. With the full logistic proliferation form, the system is no longer competitive and the mathematical analysis is carried out using an approach developed by Li and Muldowney [12]. More recently, Wang [11] studied system (1) with a Holling type-II incidence form. Local and global stability analysis are carried out.

In this paper, we will focus on system (1) with a ful-

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‡Manuscript received January 1, 2011; revised April 1, 2011.