

# A NEW TRAJECTORY GENERATION AND TRACKING CONTROL DESIGN FOR NONHOLONOMIC MOBILE ROBOTS USING COMPLEX POTENTIALS

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**Abstract.** In this paper, we present an analytical solution for the trajectory generation and tracking control design of mobile robots with nonholonomic constraints moving in a dynamic environment populated with obstacles. By explicitly defining complex potential primitives, the desired trajectory from a start position to the goal position can be generated following the negative gradient of complex potential. The proposed design yields no local minimum, and is feasible for the tracking control design of mobile robots with nonholonomic constraints. Numerical examples are provided to illustrate the proposed method.

**Keywords.** Trajectory generation, nonholonomic mobile robot, complex potential, obstacle avoidance, tracking control

## 1 Introduction

Autonomous control of mobile robots has attracted a lot of research interests due to its potential in real-world applications as seen in the missions such as search and rescue, information acquisition in hazardous environments, industry for factory floor automation, and so on. Fundamentally, the control problem of mobile robots exploring or moving within a dynamic environment boils down to make the robot move from the starting point to the target point while avoiding collisions with obstacles in the working environment. Therefore, it is of paramount importance to design real-time trajectory generation and trajectory tracking control algorithms in order to make the robot move autonomously in a dynamically changing environment.

While there exist plenty of methods for the trajectory planning of holonomic mobile robots in the structured and/or unstructured environment, such as that potential field [17], vector field histogram [11],  $A^*$  algorithm, and  $D^*$  algorithm [14], the issue of trajectory planning and control for nonholonomic mobile robots has been

known as a difficult one, since the inherent kinematic constraints associated with mobile robot system dynamics make time derivatives of some configuration variables non-integrable, and thus a collision-free path in the configuration space may not be achievable, and in general the standard methods for the trajectory planning are not directly applicable to the trajectory generation and control of mobile robots with nonholonomic constraints. An intuitive example can be found in the parallel parking experience of a car.

In this paper, we present a two-stage approach to address the motion planning and control problem for nonholonomic mobile robots moving in an uncertain environment. That is, a desired closed-loop trajectory is first generated by using complex potential method in real time and then the trajectory tracking control law is designed by explicitly taking into account the nonholonomic constraints of the robot dynamics. The application of harmonic potential function to motion planning was pioneered by the work in [1, 2], in which harmonic function was used to address the local minima problem with standard potential field method [11]. Nonetheless, the harmonic function is only solved numerically by using finite difference approach. In [18], analog resistive networks were created to obtain the electric potential subject to Neumann boundary conditions which is then used to generate the collision free path. To improve the computation efficiency and real time applicability, analytical solutions to harmonic functions were subsequently pursued in [4–6, 9] [12, 21]. In particular, the panel method was applied for the construction of harmonic function based on the shape of obstacles in [4, 9, 12]. However, the panel strength has to be carefully selected in order to obtain the solution. In [5, 6], harmonic functions were computed by putting a positive charge in the security circle center and a negative charge in the goal center. In [21], stream function approach was used for the vehicle motion planning amidst circular obstacles.

In this paper, motivated by the work in [21], we propose an analytical solution by using the concept of complex potential for trajectory generation problem of mobile robots in an environment with arbitrary shaped obstacles. Specifically, based on circle theorem [13, 21] and using

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