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STABILITY OF SOLUTIONS TO NONLINEAR NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS IN BANACH SPACES

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Abstract. In this paper, we present the recent work of the authors in the field of stability of nonlinear neutral functional differential equations (NFDEs). We first recall the stability, contractivity and asymptotic stability results of the solution to nonlinear NFDEs. As an illustration of the application of these results, the stability and asymptotic stability results of the solution to neutral delay differential equations (NDDEs) and neutral delay integro-differential equations (NDIDEs) are obtained.

Keywords. Neutral functional differential equations, neutral delay differential equations, neutral delay integro-differential equations, stability, contractivity, asymptotic stability, Banach spaces.
AMS subject classification. 34K40, 34K20, 65L05

1 Introduction

let $\mathbf{X}$ be a real or complex Banach space with the norm $\|\cdot\|$. For any given integer $q \geq 0$ and real numbers t_1, t_2, let the symbol $C^q_{\mathbf{X}}[t_1,t_2]$ denote a set consisting of all q times continuously differentiable mapping $x : [t_1,t_2] \to \mathbf{X}$, on which the norm is defined by

$$x \in C^q_{\mathbf{X}}[t_1,t_2], \qquad \|x\|_{C^q_{\mathbf{X}}[t_1,t_2]} = \sum_{i=0}^{q} \sup_{t\in[t_1,t_2]} \|x^{(i)}(t)\|.$$

Especially, $C^0_{\mathbf{X}}[t_1,t_2]$ will be simply denoted by $C_{\mathbf{X}}[t_1,t_2]$. For any given real numbers t_1, t_2,the symbol $Q_{\mathbf{X}}[t_1,t_2]$ denotes a space consisting of all everywhere continuous mapping x on $[t_1,t_2]$ except, possibly, for a finite set of points of discontinuity of the first kind (at which x is continuous on the right), with the norm

$$x \in Q_{\mathbf{X}}[t_1,t_2], \qquad \|x\|_{Q_{\mathbf{X}}[t_1,t_2]} = \sup_{t\in[t_1,t_2]} \|x(t)\|.$$

Consider neutral functional differential equations (NFDEs)

$$y'(t) = f(t, y(t), y(\cdot), y'(\cdot)), \quad t \in I_T = [t_0, T] \tag{1}$$

subject to

$$y(t) = \phi(t), \quad t \in I_\tau = [t_0 - \tau, t_0], \tag{2}$$

where $t_0,\ \tau,\ T$ are constants, $-\infty < t_0 < T < +\infty$, $0 \leq \tau \leq +\infty$, $f : [t_0,T]\times\mathbf{X}\times C_{\mathbf{X}}[t_0-\tau,T]\times Q_{\mathbf{X}}[t_0-\tau,T] \to \mathbf{X}$ is a given continuous mapping, $\phi \in C^1_{\mathbf{X}}[t_0-\tau,t_0]$. Notice that as special cases of equations (1), we have:

(i) Functional differential equations (FDEs)

$$y'(t) = f(t, y(t), y(\cdot)), \quad t \geq t_0; \tag{3}$$

(ii) Neutral delay differential equations (NDDEs)

$$\begin{aligned} y'(t) &= g\,(t, y(t), y(\eta_1(t)), y(\eta_2(t)), \cdots, y(\eta_r(t)), \\ &\quad y'(\zeta_1(t)), y'(\zeta_2(t)), \cdots, y'(\zeta_s(t)))\,, \quad t \geq t_0; \end{aligned} \tag{4}$$

(iii) Neutral delay integro-differential equations (NDIDEs)

$$y'(t) = g\left(t, y(t), y(\zeta(t)), y'(\zeta(t)), \int_{\eta_1(t)}^{\eta_2(t)} K(t,\xi,y(\xi),y'(\xi))d\xi\right), \quad t \geq t_0; \tag{5}$$

(iv) Neutral delay differential equations with maxima (NDDEMs)

$$y'(t) = f\left(t, y(\eta_0(t)), \max_{h\leq s\leq\eta_1(t)} y(s), y'(\zeta_0(t)), \max_{h\leq s\leq\zeta_1(t)} y'(s)\right), \quad h \leq \eta_i(t), \zeta_i(t) \leq t, \quad i = 0, 1; \tag{6}$$

and many others.

Remark 1.1. In the literature on the theoretical analysis of NFDEs, NFDEs is generally written as the following form (see, for example, [8, 11]):

$$\begin{cases} \dot{y}(t) = f(t, y(t), y_t, \dot{y}_t), & t \in I_T = [t_0, T], \\ \quad y_t(\theta) = y(t+\theta), & -\tau \leq \theta \leq 0, \\ y_{t_0} = \phi, & \dot{y}_{t_0} = \dot{\phi}, \end{cases} \tag{7}$$

where $\tau > 0$, $\dot{y}_t := (\dot{y})_t$. But in the literature on the numerical solutions of NFDEs, NFDEs is generally written as the form (1) (see, for example, [1, 13–16]). So in this paper, we will use the form (1).

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