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SYNCHRONIZATION OF A CONTROLLED, NOISED, GYROSCOPE SYSTEM WITH AN EXPECTED GYROSCOPE SYSTEM

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Abstract. In this paper, under a control law, the synchronization of a controlled, noised gyroscope system with an expected gyroscope system are discussed when the expected gyroscope system experiences periodic motions and the noised gyroscope system is chaotic motion. Based on the theory of discontinuous dynamical systems, the necessary and sufficient conditions for such synchronization are developed. Based on such necessary and sufficient conditions, non-synchronization, partial and full synchronizations of the controlled, noised, gyroscopes system with the expected gyroscope system are discussed. The switching scenarios between desynchronized and synchronized states of the two dynamical systems are presented, and the parameter map for such synchronizations is achieved. This study will provide an efficient way to control the noised gyroscope system in practical application. The control law is very simple and the controller can be easily implemented.

Keywords. gyroscope systems, noised gyroscope systems, discontinuous dynamical system, full synchronization, partial synchronization.

AMS subject classification. 34D06, 37B55, 37N35.

1 Introduction

The gyroscope system is extensively applied in navigation, aeronautics and space. Leipnik and Newton [1] first presented the chaotic motion in a heavy, symmetric gyroscope. In 1996, the nonlinear dynamics of a symmetric heavy gyroscope mounted on a vibrating base was investigated in Ge et al [2], and the chaotic motions of the gyroscope system with linear damping were discussed. Chen [3] explored the dynamic behaviors of a periodically forced, symmetric gyroscope system with a nonlinear damping. Periodic and chaotic motions for such a system are investigated by Poincare maps, Liapunov exponents and bifurcation diagrams. Van Dooren [4] shows the symmetric responses of such a gyroscope system. In Chen [3], chaos synchronization of two identical symmetric gyroscope systems was investigated through four couplings. The coupling strengths of the two identical systems were determined using the Lyapunov exponents for the master and slave systems. Since then, chaos synchronization for such gyroscope system with the nonlinear damping has been studied. For instance, Lei et al [5] used active control technique to synchronize two nonlinear gyroscope systems, and the sufficient conditions for global asymptotic synchronization were achieved. The adaptive sliding mode control was used for synchronization of two similar gyroscope systems with unknown parameters (e.g., [6], [7]). Salarieh and Alasty [8] applied the sliding control to synchronize two chaotic gyroscope systems with stochastic base excitation. Yau [9] proposed a fuzzy sliding mode control to synchronize two chaotic nonlinear gyroscope systems with uncertainties and external disturbances. In the aforementioned studies, all the control laws are very complicated, which make the implementation be difficult. The adopted techniques cannot give necessary and sufficient conditions, and the asymptotic stability is determined based on the Lyapunov direct method. Thus, only the asymptotic synchronization can be obtained. However, Luo [10] proposed a new theory for the synchronization of dynamical systems, and this theory provides an alternative way to investigate system synchronization. Such a theory for dynamical systems synchronization with specific constraints is based on the theory of discontinuous dynamical systems, which can be referred to Luo [11-13]. In such a theory, the G-functions are a key to determine the switchablity of a flow from one domain to another in discontinuous dynamical systems.

In this paper, the synchronization of the noised gyroscope system with the expected gyroscope system will be investigated through the theory of discontinuous dynamical system. The expected gyroscope system experiences periodic motions, and the noised gyroscope system is chaotic motion. Under a control law, the synchronization mechanism of the controlled, noised gyroscope system with the expected systems will be discussed. Based on the theory of discontinuous dynamical systems, the necessary and sufficient conditions for such synchronization will be developed. Based on such necessary and sufficient conditions, non-synchronization, partial and ful-

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