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FLUCTUATING NUTRIENT INPUT IN A SIMPLE PLANKTON SYSTEM

S. Gakkhar, A. Priyadarshi and Sandip Banerjee *†‡§

Abstract. An aquatic model consists of nutrient, phytoplankton and zooplankton has been analyzed in this paper. The system is proved to be dissipative in non-negative octant. A criteria for persistence of all species in the system with constant nutrient input has been established. Extensive numerical investigations reveal variation in amplitude and frequencies of the periodic orbits for sensitive parameters. In case of fluctuating nutrient input, system exhibits complex behavior.

Keywords. Non-linear Mortality, Periodic Nutrient Input, Bifurcation Analysis, Hopf-Points, Limit Orbits.

1 Introduction

After the pioneer work of Riley [19], several dynamical models of different complexities have been investigated for phytoplankton-zooplankton interactions [22, 23]. The importance of nutrient to growth of these two species leads to its explicit incorporation in the models, popularly known as NPZ models [2, 3, 4, 5, 6, 10, 24, 25, 26, 28]. Aquatic models of plankton food chains are usually assembled with three components

(i) source terms that describe the inputs of nutrients or the dynamics of basal species, which do not feed on other species in the chain;

(ii) consumption terms that link consumers and their resources within the chain;

(iii) Closure terms describing the mortality of top predators, which are not fed upon by other species in the chain, acknowledging their role in truncating the food web [24, 25, 28].

Consider an open system with three interacting components consisting of dissolved limiting nutrient (N), phytoplankton (P) and herbivorous zooplankton (Z). The following three coupled ordinary differential equations gives the expression for the model:

$$\begin{aligned}\frac{dN}{dT} &= -g(N)\mu P + \beta h(P)Z + i(P)P + j(Z)Z \\ &+ k(N_{input} - N) \qquad (1)\\ \frac{dP}{dT} &= P[g(N)\mu - i(P) - (s+k)] - h(P)Z \\ \frac{dZ}{dT} &= Z[\alpha h(P) - j(Z)]\end{aligned}$$

The nutrient decreases due to uptake by phytoplankton. μ is the maximum specific growth rate of phytoplankton. The function $g(N)$ represents nutrient uptake of phytoplankton. The specific phytoplankton growth rate is limited by nutrient availability. It is a continuous function defined on $[0,\infty)$ such that

$$g(0) = 0, \ \frac{dg}{dN} > 0 \text{ and } \lim g(N) = 1 \ .$$

In particular, this kind of function includes Holling type II or the Michaelis-Menten function

$$g(N) = \frac{N}{k_1 + N}$$

The constant k_1 is the half-saturation constant or Michaelis-Menten constant.

The zooplankton grazing response function $h(P)$ is a continuous function defined on $[0,\infty)$ such that

$$h(0) = 0, \ \frac{dh}{dP} > 0$$

In particular, a sigmoidal Holling type-III functional response formulation has been considered for grazing function of zooplankton:

$$h(P) = \frac{\lambda P^2}{{k_2}^2 + P^2}$$

Here, λ is the maximum zooplankton grazing rate and k_2 is the half saturation constant.

The term $k(N_{input} - N)$ models exchange of nutrients with the water below the mixed layer. N_{input} is the sub-mixed-layer nutrient concentration. The exchange rate k defines the fraction of the mixed layer which is exchanged daily with the deeper water.

*Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee-247667, India.

†E-mails: sungkfma@iitr.ernet.in, anupam240@gmail.com, sandofma@iitr.ernet.in

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