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SYMMETRY REDUCTIONS AND TRAVELLING WAVE SOLUTIONS FOR A HIGHER ORDER WAVE EQUATION OF KDV TYPE

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Abstract. In this work we study a higher order wave equation of KdV type from the point of view of the theory of symmetry reductions in partial differential equations. We obtain the classical symmetries admitted, then, we use the transformations groups to reduce the equations to ordinary differential equations. Physical interpretation of these reductions and some exact solutions are also provided. Among them we obtain a solitary wave solution. In [3] (see also [5]) a general theorem on conservation laws for arbitrary differential equation which does not require the existence of Lagrangians has been proved by N.H. Ibragimov. This new theorem is based on the concept of adjoint equations for non-linear equations. We prove that the higher-order KdV equation is quasi-self-adjoint. Moreover by using a general theorem on conservation laws proved in [5, 6] we derive some nontrivial conservation laws.
Keywords. Lie symmetries; Exact solutions; Travelling waves; Self-adjointness; Conservation laws.

1 Introduction

In the last century many researchers considered the one-dimensional motion of solitary waves of inviscid and incompressible fluids [15]. In this subject probably one of the most important results was the derivation of the famous KdV equation by Korteweg and de Vries [8] in 1895.

The classical KdV equation models weakly nonlinear unidirectional long waves, and arises in various physical contexts. It represents an approximation in the study of long wavelength, small amplitude inviscid and incompressible fluids. If one allows for the appearance of higher-order terms a more complicated equation is obtained, which is non-integrable but still admits some special wave solutions [10]. This equation,

$$\begin{aligned} &u_t + ku_x + \alpha uu_x + \beta u_{xxx} + \alpha^2\rho_1 u^2 u_x \\ &+\alpha\beta(\rho_2 uu_{xxx} + \rho_3 u_x u_{xx}) = 0 \end{aligned} \tag{1}$$

which will be referred to as generalized KdV, was studied in [12] by Fokas, who presented a local transformation connecting it with an integrable partial differential equation (PDE). The higher order wave equations of KdV type model strongly nonlinear long wavelength and the short amplitude waves. It is a just reason for the strongly nonlinear character and integrability of these equations attracting many researchers to study them. In [11] the authors derived analytical expressions for solitary wave solutions for some special sets of parameters and they carried out a detailed numerical study of these solutions using a Fourier pseudospectral method combined with a finite-difference scheme. In [12] the integral bifurcation method was used to study equation (1) and some new travelling wave solutions with singular or nonsingular character were obtained for some special sets of parameters.

In [5, 6] a general theorem on conservation laws for arbitrary differential equation which does not require the existence of Lagrangians has been proved. This new theorem is based on the concept of adjoint equations for non-linear equations. The notion of self-adjoint equations has been also extended to non-linear equations. Due to the fact that many equations having remarkable symmetry properties and physical significance are not self-adjoint. Therefore one cannot eliminate the nonlocal variables from conservation laws of these equations by setting v = u. In [6] the concept of self-adjoint equations has been generalized by introducing the definition of quasi-self-adjoint equations.

In this work, we study equation (1) from the point of view of the theory of symmetry reductions in partial differential equations. By using this method we bring out the unexplored invariance properties and similarity reduced ordinary differential equations (ODE's) of the above equation (1). We will also determine, for (1), the subclass of equations which are self-adjoint and quasi-self-adjoint. Moreover, by using the general theorem on conservation laws proved in [5, 6] we derive some nontrivial conservation laws for (1).

2 Classical symmetries

In this section we perform Lie symmetry analysis for equation (1). Let us consider a one-parameter Lie group

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