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MONOTONICITY AND ASYMPTOTIC BEHAVIOR OF SOLUTIONS FOR SECOND-ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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Abstract. In this paper the monotonicity and asymptotic behavior of solutions for a class of second-order nonlinear differential equations are discussed, all solutions are classified into four disjoint classes, and necessary and sufficient conditions are established for the existence of solutions in each class.

Keywords. Nonlinear differential equations, second order, monotonicity, asymptotic behavior, fixed-point theorem.

1 Introduction

The second-order nonlinear differential equation

$$[p(t)h(x(t))f(x'(t))]' = q(t)g(x(t)), \quad t \geq a, \tag{1}$$

has many applications in different areas such as Thomas-Fermi equations and Schrödinger-Persico equations in the study of atomic fields in physics, chemical reacting systems in chemical engineering, law of angular momentum conservation in celestial mechanics, and positive radial solutions of reaction-diffusion problems in partial differential equations; see [1, 5, 7, 8, 11, 20].

Clearly, if $f(r) = \Phi_p(r) := |r|^{p-2}r$, $p > 1$, the so-called p-Laplacian operator, $h(r) = 1$, and $g(r) = \Phi_\beta(r)$, Equation (1) becomes the well-known Emden-Fowler equation

$$[p(t)\Phi_p(x'(t))]' = q(t)\Phi_\beta(x(t)), \quad t \geq a, \tag{2}$$

which has been extensively studied by many researchers; see, e.g., [2, 3, 4, 6, 9, 10, 12, 13, 14, 15, 16, 17] and references cited therein.

For the case of general nonlinear functions $f(r)$, [18] studied the monotone, continuablity, boundedness, and asymptotic behavior of solutions of the differential equation

$$[p(t)f(x'(t))]' = q(t)g(x(t)), \quad t \geq a. \tag{3}$$

Recently, [19] discussed the continuability and boundedness of solutions of (1). In this paper we continue the study of (1) along the direction of [18, 19] and show that all solutions of (1) are eventually monotonic and can be divided into four classes A_b, A_∞, B_b, and B_0. Moreover, we establish necessary and sufficient conditions for the existence of solutions in each class. The obtained results in this paper have generalized and improved some analogous ones existing in the literature [3, 4, 12, 13, 18].

Throughout the paper the following assumption is imposed:

(H)

$$\begin{cases} p(t),\ q(t) : [a,\infty) \to \mathbb{R} \text{ are continuous and} \\ \quad p(t) > 0 \text{ and } q(t) > 0; \\ h(r) : \mathbb{R} \to \mathbb{R} \text{ is continuous and } h(r) > 0; \\ g(r) : \mathbb{R} \to \mathbb{R} \text{ is continuous and } rg(r) > 0 \\ \quad \text{for } r \neq 0; \\ f(r) : \mathbb{R} \to \mathbb{R} \text{ is continuous, increasing, and} \\ \quad rf(r) > 0 \text{ for } r \neq 0. \end{cases}$$

The paper is organized as follows: Section 1 is the introduction. The assumptions, solution classifications, and citations of known results are presented in this section. After that, necessary and sufficient conditions for the existence of solutions in each class are provided in Section 2 and Section 3, respectively.

Definition 1.1. A solution of (1) is a differentiable function $x(t)$ such that $p(t)h(x(t))f(x'(t))$ is differentiable and satisfies (1) on its maximum existence interval $[a, \alpha_x)$, $a < \alpha_x \leq \infty$.

Notice that we consider only solutions of (1) which are not eventually identically equal to zero.

To discuss the asymptotic behavior of solutions we need the following assumptions:

(H1) There exists a $M > 0$ such that

$$|f^{-1}(uv)| \leq M|f^{-1}(u)||f^{-1}(v)|, \quad \forall u, v \in \mathbb{R}.$$

(H2) There exists a $m > 0$ such that

$$(u - v)(g(u) - g(v)) \geq 0, \quad \forall u, v : |u| \geq m,\ |v| \geq m.$$

(H3) There exists a $r_0 > 0$ such that

$$\int_{r_0}^{\infty} \frac{dr}{f^{-1}(z(r))} = \infty, \quad \int_{-\infty}^{-r_0} \frac{dr}{f^{-1}(z(r))} = -\infty,$$

where $z(r) := g(r)/h(r)$.

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