Journal of Nonlinear Systems and Applications (2012) 64-72
http://www.watam.org/JNSA/

LPV APPROACH FOR THE STABILIZATION OF A CLASS OF NONLINEAR DYNAMICAL SYSTEMS

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Abstract. In this note, we investigate the problem of $\mathcal{H}_\infty$ observer-based state feedback controllers for a class of Lipschitz nonlinear systems. A simple systematic and useful synthesis method is proposed. Thanks to the use of the Differential Mean Value Theorem (DMVT), separation results are obtained. The synthesis conditions are given in term of Linear Matrix Inequalities (LMIs), easily tractable by convex optimization algorithms. Based on some mathematical tools such as Barbalat's lemma and convergence of series, an asymptotic convergence towards zero is provided in the $\mathcal{H}_\infty$ free context. Two numerical examples are given in order to show performances of the proposed method.

Keywords. Nonlinear observers, observer-based control, $\mathcal{H}_\infty$ analysis, LMI approach, the Differential Mean Value Theorem (DMVT), Linear Parameter Varying (LPV) systems.

1 Introduction

Tremendous research activities on the problem of implementation and observers based control for linear and nonlinear dynamic systems show a growing interest in control theory area during the last decade. Thus, compliance with specific performances requires a careful choice of controller. To do this, the ideal strategy would be to have completely states and inputs of the system. However, for intrinsic reasons to the system or for high costs of the sensors implementation, the measurement of the state is often only partial. Moreover, the measurements may be affected by noises. That is why the controllers proposed in the literature are often based on state observers. Indeed, the observers incorporate a filter that estimates the entire state vector, or the unmeasured part only. It is one of the main reasons for which the observers design problem is extensively investigated in the recent literature [1], [2], [3], [4], [5], [6]. However, very less approaches concern the observer-based control problem for nonlinear systems. In the linear case, the problem is easy to investigate thanks to the separation principle. Nevertheless, the main difficulty for nonlinear systems lies in the fact that the separation principle is not always true.

Consequently, obtaining synthesis conditions in term of LMIs is a difficult task because of bilinear couplings between certain variables resulting from the stability analysis. To overcome this obstacle, LMI conditions under an equality constraint for a class of nonlinear discrete time systems are established [7]. But, this equality constraint is difficult to be solved for certain systems, in particular those with a single input.

Till now, the problem of observer-based state feedback controllers for nonlinear systems remains an open research subject. Obtaining a systematic observer-based control method for nonlinear systems under LMI conditions becomes a difficult challenge.

It is worth to notice that several research activities have been paid toward the study of the problem of observer-based control for linear systems and many applications are provided for the nonlinear case. We refer the reader to [8], [9], [10], [11], [12], just to mention some works. Note also that separation results are proposed in [13] and [14] for a class of nonlinear systems using a sufficiently fast high-gain observer. An extension, by the same authors, is given in [15], where their proposed state feedback controller renders a certain compact set positively invariant and asymptotically attractive. On the other hand, for all these works recent advances on stability analysis, using contraction theory, may be investigated [16], [17].

This paper deals with the problem of $\mathcal{H}_\infty$ observer-based state feedback controllers for a class of Lipschitz nonlinear systems. A simple systematic synthesis method is proposed. Thanks to the use of the DMVT, we obtain separation results for a class of nonlinear systems. The synthesis conditions are given in term of linear matrix inequalities, easily tractable by convex optimization algorithms. It should be noticed that this work concerns the continuous-time and the discrete-time systems. Indeed, the main contribution of this work may be summarized into two points. The first one lies in the fact that a unified observer based control approach is established for both continuous and discrete time systems. The second point concerns the stability conditions which are expressed in terms of LMIs with $\mathcal{H}_\infty$ performances.

This note is arranged as follows. In section 2, we state the problem formulation that we consider in this study.

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