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TRANSFER ENTROPY ON RANK VECTORS

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Abstract. Transfer entropy (TE) is a popular measure of information flow found to perform consistently well in different settings. Symbolic transfer entropy (STE) is defined similarly to TE but on the ranks of the components of the reconstructed vectors rather than the reconstructed vectors themselves. First, we correct STE by forming the ranks for the future samples of the response system with regard to the current reconstructed vector. We give the grounds for this modified version of STE, which we call Transfer Entropy on Rank Vectors (TERV). Then we propose to use more than one step ahead in the formation of the future of the response in order to capture the information flow from the driving system over a longer time horizon. To assess the performance of STE, TE and TERV in detecting correctly the information flow we use receiver operating characteristic (ROC) curves formed by the measure values in the two coupling directions computed on a number of realizations of known weakly coupled systems. We also consider different settings of state space reconstruction, time series length and observational noise. The results show that TERV indeed improves STE and in some cases performs better than TE, particularly in the presence of noise, but overall TE gives more consistent results. The use of multiple steps ahead improves the accuracy of TE and TERV.

Keywords. bivariate time series, coupling, causality, information measures, transfer entropy, rank vectors.

1 Introduction

The fundamental concept for the dependence of one variable Y measured over time on another variable X measured synchronously is the Granger causality [1]. While Granger defined the direction of interaction in terms of the contribution of X in predicting Y, many variations of this concept have been developed, starting with linear approaches in the time and frequency domain (e.g. see [2, 3]) and extending to nonlinear approaches focusing on phase or event synchronization [4, 5, 6], comparing neighborhoods of the reconstructed points from the two time series [7, 8, 9, 10, 11, 12, 13], and measuring the information flow between the time series [14, 15, 16, 17, 18].

Among the different proposed measures we concentrate here on the last class of measures, and particularly on the transfer entropy (TE) [14] and the most recent variant of TE operating on rank vectors, called symbolic transfer entropy (STE) [17] (see also [19] for a similar measure). There have been a number of comparative studies on information flow measures and other coupling measures giving varying results. In all the studies where TE was considered, it performed at least as good as the other measures [20, 21, 22]. The STE measure is proposed as an improvement of TE in real world applications, where noise may mask details of the fine structure, that can be better treated by coarse discretization using ranks instead of samples.

We propose here a correction of STE. In the definition of TE the observable of the response at one time step ahead is a scalar, but in STE it is taken as a rank vector at this time index. We modify STE to conform with the definition of TE and give grounds for the correctness of this modification. Further, we allow for the future of the response to be defined over more than one time steps. To the best of our knowledge this has not been implemented in TE, but it is the core element in the information flow measures of mean conditional mutual information [18] and coarse-grained transinformation rate [15]. In many applications on interacting flow systems, the sampling time may be small and a single step ahead may not regard the time of the response to the directed coupling, as in electroencephalography (EEG) [17, 23], and in the analysis of financial indices [16, 24]. The same may hold for maps: the transfer of information may better be seen over more than one iteration of the interacting maps. We compare TE, STE and our correction of STE on measuring weak directed interaction in some known coupled systems and for different state space reconstructions, time series lengths and also in the presence of noise. We also investigate the change in the performance of these measures when defining them for more than one step ahead.

In the following, TE and STE measures are presented briefly in Section 2, and the proposed modification of TE is described in Section 3. Then the results of the simulation study comparing the proposed measure to TE and STE are presented in Section 4, and discussed in Section 5.

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