## DETERMINISTIC AND STOCHASTIC NEUTRAL SYSTEMS ON BANACH SPACES AND THEIR OPTIMAL FEEDBACK CONTROLS\*

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Abstract. In this paper we consider a class of second order neutral differential equations on infinite dimensional Banach spaces. The system is driven by vector measures considered as control variables. We prove existence of optimal controls for Bolza problem and time optimal control problems. Also we consider stochastic versions of the system. We prove existence and regularity of solutions and consider certain problems of optimal control of measures induced by them. We consider two types of controls: vector measures as open loop controls, and bounded linear operators as feedback controls and prove existence of optimal controls. Some open problems are indicated.

**Keywords.** Neutral systems, Deterministic and stochastic, Banach spaces, Vector Measures, Optimal Control, Operators as feedback control.

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<sup>\*</sup>JOURNAL OF NONLINEAR SYSTEMS AND APPLICA-TIONS, VOL. 4, NO. 1, PP. 1-10, 2013.

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 $<sup>^{\</sup>ddagger}7\mathrm{th}$  DEDS Conference, University of Southern Florida, Dec. 15-18, 2010.

<sup>&</sup>lt;sup>§</sup>Manuscript received October, 2011; accepted August, 2012.