

STOCHASTIC SWITCHING RELIABLE CONTROL FOR A CLASS OF STOCHASTIC HYBRID SYSTEMS WITH STATE DELAY

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Abstract. This paper deals with the design of a switching reliable control for a class of stochastic hybrid system with constant delay and nonlinear lumped uncertainties having linear-growth-type bounds. Two sets of actuators are considered; namely, a set of operational actuators that never fail, and a set of actuators that are susceptible to failure. The main objective here is to design a state feedback controller for each system mode such that, for all admissible nonlinear uncertainties and actuator failures occurring in a pre-specified subset of actuators, the closed-loop modes are exponential stable in the mean square (m.s.). Moreover, to maintain the stability property of the switched system, a dwell-time-type switching rule and the technique of multiple Lyapunov function together with the Razumikhin methodology are used. This approach leads to solving a set of algebraic Riccati-like equations. Finally, a numerical example with simulations is presented to illustrate the usefulness of the proposed control design.

Keywords. Reliable control, stochastic switching systems, time delay, multiple Lyapunov functions, Razumikhin methodology.

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‡Manuscript received September 2012; revised August 2013.

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