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MATHEMATICAL ANALYSIS OF CHOLERA EPIDEMIC MODEL WITH SEASONALITY*

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Abstract. In this paper, we conduct a mathematical analysis of the cholera model proposed in [20] in the case of non periodic and periodic contact rate $\beta(t)$. We study the stability of equilibria and show that there is always a globally asymptotically stable equilibrium state. Depending on the value of the basic reproduction ratio R_0, this state can be either endemic ($R_0 > 1$), or infection - free ($R_0 < 1$). We demonstrate a real-world application of this model by investigating the recent cholera outbreak in Cameroon. Meanwhile, we present numerical results to verify the analytical prediction.

Keywords. cholera epidemics, dynamical system, Equilibrium, Stability, basic reproduction number.

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