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SLOW INVARIANT MANIFOLD OF HEARTBEAT MODEL*

Jean-Marc GINOUX and Bruno ROSSETTO †‡

Abstract. A new approach called *Flow Curvature Method* has been recently developed in a book entitled *Differential Geometry Applied to Dynamical Systems*. It consists in considering the trajectory curve, integral of any n-dimensional dynamical system as a curve in Euclidean n-space that enables to analytically compute the curvature of the trajectory - or the flow. Hence, it has been stated on the one hand that the location of the points where the curvature of the flow vanishes defines a manifold called *flow curvature manifold* and on the other hand that such a manifold associated with any n-dimensional dynamical system directly provides its *slow manifold analytical equation* the invariance of which has been proved according to Darboux theory. The *Flow Curvature Method* has been already applied to many types of autonomous dynamical systems either singularly perturbed such as Van der Pol Model, FitzHugh-Nagumo Model, Chua's Model, etc. or non-singularly perturbed such as Pikovskii-Rabinovich-Trakhtengerts Model, Rikitake Model, Lorenz Model,... Moreover, it has been also applied to non-autonomous dynamical systems such as the Forced Van der Pol Model. In this article it will be used for the first time to analytically compute the *slow invariant manifold analytical equation* of the four-dimensional Unforced and Forced Heartbeat Model. Its *slow invariant manifold equation* which can be considered as a "state equation" linking all variables could then be used in heart prediction and control according to the strong correspondence between the model and the physiological cardiovascular system behavior.
Keywords. flow curvature method, slow invariant manifold analytical equation, heartbeat Model.

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†Jean-Marc GINOUX and Bruno ROSSETTO are with Aix Marseille Université, CNRS, ENSAM, LSIS, UMR 7296, 13397 Marseille, France and Université de Toulon, CNRS, LSIS, UMR 7296, B.P. 20132, 83957, La Garde Cedex France E-mail: ginoux@univ-tln.fr

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