

# MATHEMATICAL ANALYSIS OF A GENERAL CLASS OF INTRA-HOST MODEL OF MALARIA WITH “ALLEE EFFECT”\*

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**Abstract.** This paper investigates the global dynamics of an intra-host model of malaria with immune effectors. We use the “Allee effect” to model the dynamics of the immune effectors. The biological consequence of the ‘Allee effect’ is that there exists a threshold value such that the immune effectors can persist in the body above this threshold. We compute the basic reproduction ratio  $\mathcal{R}_0$ . We show that the merozoite-free equilibrium is locally asymptotically stable when  $\mathcal{R}_0 \leq 1$ , while when  $\mathcal{R}_0 > 1$ , the merozoite-free equilibrium is unstable and there exists a unique endemic equilibrium which is locally asymptotically stable under some conditions. Moreover, we show that the model can undergo a Hopf bifurcation at the positive equilibrium and exhibit periodic oscillations. Numerical simulations are provided to illustrate analytical results.

**Keywords.** Malaria; intra-host models; immune effectors; “Allee effect”; Stability; Hopf bifurcation.

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