

HYBRID SYNCHRONIZATION BETWEEN DIFFERENT FRACTIONAL ORDER HYPERCHAOTIC SYSTEMS USING ACTIVE CONTROL METHOD*

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Abstract. In this article the hybrid synchronization between two different fractional order hyperchaotic systems viz., Lorenz system and Newton-Leipnik system is studied. Based on stability theory, the sufficient conditions for achieving the hybrid synchronization of two hyperchaotic systems are derived. The active control method is very effective and convenient to achieve hybrid chaos synchronization of the non-identical chaotic systems. The effects on synchronization time due to the presence of fractional order derivatives are the key features of the present article. Numerical simulation results which are carried out using Adams-Boshforth-Moulton method show that the method is effective and convenient for synchronization of fractional order dynamical systems.

Keywords. Chaos; synchronization; Fractional order derivative; Hyperchaotic Lorenz and Newton-Leipnik systems.

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