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COMPLEX DYNAMICS OF FOOD CHAIN MODELS AND BIOLOGY OF THE TOP PREDATOR

Ranjit Kumar Upadhyay, Parimita Roy, S.K. Tiwari *†

Abstract. An attempt has been made to understand how the dynamics of food chain models depend on biology of the top predator. In this paper, two three-species food chain models with specialist and generalist top-predators has been proposed and analyzed. The proposed model systems are the extension of a well-known Truscott and Brindley model [28] in aquatic environment. We have carried out the existence and stability analysis of the equilibria of the model systems using Routh Hurwitz criterion and Lyapunov direct method. We investigate the dynamical transitions in the model system by treating different parameters appearing in the model as control parameter. We identify the existence of Hopf-bifurcation and demonstrate the period-doubling route to chaos when the ratio of per capita growth rate of predator population with the intrinsic growth rate of prey population, is treated as bifurcation parameter. Chaotic dynamics is depicted by 2D parameter scans, basin boundary calculations and bifurcation analysis. Chaotic attractors with riddled basin and fractal boundaries are also observed. Our studies show that both the model system exhibits deterministic chaos when some control parameters attain their critical values; but the generating mechanisms are different.

Keywords. Food- chain model .Top predator. Period-doubling route. Hopf-bifurcation. Chaotic attractor.

1 Introduction

Food-chain is a reality in nature. The dynamics of food-chain/food web models has long been and will continue to be one of dominant topics in mathematical ecology. Prey-predator interaction is well known for generating oscillatory dynamics and its mechanisms were successfully explained by Lotka and Volterra. Many attempts have been made to apply Lotka Volterra model to real world oscillatory dynamics. May and Leonard [1] demonstrated that systems of cyclic competition could also develop chaotic oscillations. There are many conceptual food chain models considering specific functional form of predator-prey interaction. Mathematical models describing food-chain system have been studied by many researchers [2]-[9]. Hastings and Powell (HP model) [2] have proposed a food-chain model system where a prey population is captured by individual members of a predator population. This predator population, in turn, serves as a favorite food for individuals of top specialist predator population. They found that chaos existed in a large parameter ranges in their model systems. McCann and Yodzis [10] obtained biological realistic conditions for chaos to exist in HP model system. McCann and Hastings [11] stabilized the food chain by eliminating chaotic dynamics and limit cycles behavior of HP model by introducing omnivore on top-predator and increasing its strength. In this paper, we have modified the HP model by replacing the Holling type II functional response with Holling type III functional response in all the three level of population assuming the vertebrate prey and predator population forming the food chain model. Xu and Li [12] showed that the deterministic tri-trophic food chain model stabilized the food chain system and leading cyclic and chaotic dynamics to a steady state. Some models may suggest greater likelihood of chaos than others and this may be due to the strength and type of interactions present. Upadhyay and Rai [13] have proposed a food-chain model system where a prey population is captured by individual members of a predator population. This specialist predator population, in turn, serves as a favorite food for individuals of top generalist predator population which very often switch to alternative food option when their favorite food is in short supply. They have made an attempt to investigate the reasons for the failure of attempts to observe chaos in real ecological system and found that chaos existed in narrow parameter ranges in their model systems. Ferriere and Gatto [14] investigated the chaotic dynamics which results from natural selection. They suggested that chaos could be an optimal behavior of various biological systems.

In [8] and [15] authors have suggested that the biology of the top-predator would play an important role for the determination of food chain dynamics. The dynamics of natural systems with specialist top predators are dictated by stochastic influence whereas with generalist top predators are governed by deterministic changes in system parameters. Food chain predation occurs when a top predator feeds on an intermediate consumer which feeds on a resource. The top predator can inhibit the growth rate

*Ranjit Kumar Upadhyay, Parimita Roy and S.K. Tiwari are with Department of Applied Mathematics, Indian School of Mines,Dhanbad, Jharkahnd-826004, India. E-mails: ranjit_ism@yahoo.com, roy.parimita@gmail.com

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