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NONLINEAR CONTROLLER DESIGN FOR CLASS OF PARABOLIC-HYPERBOLIC SYSTEMS

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Abstract. This contribution concerns the problem of finite dimensional control for a class of systems described by nonlinear hyperbolic-parabolic coupled partial differential equations (PDE's). Initially, Galerkin's method is applied to the PDE system to derive a nonlinear ordinary differential equation (ODE) system that accurately describes the dynamics of the dominant (slow) modes of the PDE system. After, we introduce a useful nonlinear controller to assure stabilization under convex sufficient conditions. At last, we give a numerical example showing the effectiveness of the proposed controller and some comments illustrate this approach.

Keywords. Radiative transfer equation, Nonlinear heat equation, Galerkin method, Linear Matrix Inequalities (LMIs).

1 Introduction

A large number of industrially important transport-reaction processes are inherently nonlinear and are characterized by significant spatial variations because of the underlying diffusion and convection phenomena. Representative examples include rapid thermal processing, plasma reactors, and crystal growth processes, to name a few. The mathematical models which describe the spatiotemporal behavior of these processes are typically obtained from the dynamic conservation equations and consist of systems of parabolic and hyperbolic partial differential equations (PDEs). The main feature of parabolic PDE systems is that the eigenspectrum of the spatial differential operator can be partitioned into a finite-dimensional slow one and an infinite-dimensional stable fast complement [4]. Motivated by this fact, a typical approach to the control design of linear or semilinear parabolic PDE systems is to obtain an approximate ordinary differential equation (ODE) representation of the original PDE system by utilizing the spatial discretization techniques, which is then used for controller design purposes by applying different existing ODE-based linear or nonlinear control techniques. The standard Galerkin method was used to derive a finite-dimensional ODE model. In general, the computation of the controller action becomes more expensive with increasing controller dimension. This is one reason why full order synthesis control has not been widely used in industry. Recently, linear matrix inequalities (LMIs) have attained much attention in control engineering [2, 6], since many control problems can be formulated in terms of LMIs and thus solved via convex programming approaches. In this note, we construct a boundary controller to establish the stability of the coupled radiative-conductive heat transfer systems in the finite dimensional. Through Lyapunov analysis we established sufficient conditions for stability by the feasibility of the finite-dimensional Linear Matrix Inequalities(LMI).

This article is organized as follows. Next section develops the governing equations for two-dimensional combined radiative and conductive heat transfer. In section 3, the reduced model of the coupled partial differential equations is given. The section 4, a linear and nonlinear controller are proposed to assure stabilization under convex sufficient conditions. Simulations results are given in section 5 and the last section draws conclusions.

Notations. Throughout this paper, we will use the following notation:

- $\|.\|$ is the Euclidean norm.
- $(\star)$ is used for the blocks induced by symmetry.
- $\mathbb{I}$ represents the identity matrix of appropriate dimension.
- $\mathbb{A}^T$ represents the transposed matrix of $\mathbb{A}$.
- For a square matrix $\mathbb{S}$, $\mathbb{S} \succ 0$ (respectively $\mathbb{S} \prec 0$) means that this matrix is positive definite (respectively negative definite).
- $e_s^T(i) = (\underbrace{0, ..., 0, \overbrace{1}^{\text{ith}}, 0, ..., 0}_{\text{s components}}) \in \mathbf{R}^s, s \geqslant 1$ is a vector of the canonical basis of $\mathbf{R}^s$.

2 Problem statement

Let Ω be a bounded domain in $\mathbf{R}^2$ and $\mathcal{D}$ the unit disk. Thus, for the problem considered, $\beta \in \mathcal{D} = \{\beta \in \mathbf{R}^2 :$

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