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DYNAMICS OF LESLIE-GOWER MODEL WITH SIMPLIFIED HOLLING TYPE IV FUNCTIONAL RESPONSE

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Abstract. In this paper a three-species of Leslie-Gower predator-prey food chain model with Holling type IV functional response is proposed. The dissipativeness of the solution of the model is discussed. Local and global stability analyses of the system are carried out. The dynamics of the predator-prey food chain model with simplified Holling type IV functional response is investigated theoretically and numerically.

Keywords. bifurcation, Holling type IV, Leslie-Gower, periodic, predator-prey.

AMS Subject Classification. 37D45, 37D50, 37C75.

1 Introduction

Around four decades after the work of May in [26] exploring chaotic behaviors in a simple model of population dynamics. This work of May inspired much research work, dealing with periodic, quasi periodic and even chaotic in predator-prey system see [8,9,10,11,12,13,14,15]. Alaoui in [1] proposed and discussed the complex dynamics of modified Leslie-Gower food chain model with Holling type II response function. Naji in [11] discussed a modified Leslie-Gower food chain model with Bendigton-DeAnglis functional response and the model exhibited chaotic dynamics. Gakkhar with Priyadarshi in [4] proposed a Leslie-Gower food web model. The numerical works of Alaoui, Naji and Gakkhar are brilliant.

The functional response of Holling type IV describes a situation in which the predator's per capita rate of predation decreases at sufficiently high prey densities see [16]. Upadhyay in [18] used Holling type IV functional response of the form $\frac{wx}{d+x+(x^2/i)}$ to investigate the existence of complex dynamics in a three species food chain model, about how this functional response obtained see [5,6,7]. Sokol and Howell in [21] suggested a simplified Holling type IV function of the form $\frac{wx}{d+x^2}$ and found that it is simpler and better than the original function of Holling type IV in their experiments about the kinetics of phenol oxidation by washed cells. Ruan in [20] and Hu in [22] are both studied the dynamics and b-ifurcation analysis of continuous-time and discrete-time models respectively. Investigations in Leslie-Gower type model see [1,2,3,4] is less than the other types like Lotka-Volterra and Bendigton-DeAnglis functional responses, also Holling type IV functional response is relatively less studied in population ecology see [18,19,20,21,22].

This paper is organized as follows: In section 2, the mathematical model is proposed and each parameter in the model is described. In section 3, the dissipativeness of the solutions of the model are established. Stability analysis of the equilibrium points of the model are derived in section 4. In section 5, numerical study are carried out to obtain the chaos, periodic and stability of the model. Finally, the paper is end with a conclusions and results in section 6.

2 The Mathematical Model

Consider the three species food chain model consisting of the prey whose population density at time t denoted by *x(t)*, the intermediate predator whose population density at time t denoted by *y(t)* and the top predator whose population density at time t denoted by *z(t)*. The intermediate predator y preys on its sole food x at the lower level according to the Holling type-IV functional response, while the top predator z preys on y at the second level according to the modified Leslie-Gower type. A typical example of this type would involve a rodent, snake, peacock food chain see [3,17].

The dynamics of the above food chain model can be represented by the following set of differential equations:

$$\begin{aligned}
\frac{dx}{dt} &= x\left(a_0 - b_0 x\right) - \frac{v_0 xy}{d_0 + x^2} : x(0) \geq 0, \\
\frac{dy}{dt} &= \frac{v_1 xy}{d_1 + x^2} - a_1 y - \frac{v_2 yz}{d_2 + y} : y(0) \geq 0, \\
\frac{dz}{dt} &= c_3 z^2 - \frac{v_3 z^2}{d_3 + y} : z(0) \geq 0. \qquad (1)
\end{aligned}$$

Here the positive constants a_0, b_0, v_0, d_0, v_1, a_1,d_1,

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