Journal of Nonlinear Systems and Applications (2016)35–43
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A SYMBOLIC DYNAMICS PERSPECTIVE OF ROCK-PAPER-SCISSOR GAME

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Abstract. The Rock-Paper-Scissor (RPS) games are investigated under the framework of symbolic dynamics. It is of interest that several Chua's nonlinear dynamics analytical methods are appropriate for the RPS game rule. And a series of dynamical properties on the its subsystems are explored by the directed graph and transition matrix. Moreover, the method presented in this paper is also applicable to other game rules.

Keywords. Rock-Paper-Scissor game; Symbolic dynamics; Lameray diagram; Characteristic function; basin tree diagram; Topologically mixing; Topological entropy.

1 Introduction

Evolutionary game theory provides an effective method for analyzing the complex behaviors in the multi-species ecological systems (see [1, 2]). As a sort of simple evolutionary game models, the Rock-Paper-Scissor (RPS) games are endowed with the fundamental non-cooperative competition of three distinct strategies. In particular, RPS game rules reflect the cyclic dominance: rock can defeat scissor, scissor conquers paper, and paper in turn competes rock. In nature, the common side-blotched lizard exhibits a RPS pattern in its mating strategies (see [3]). By the general evolutionary models, some bacteria also exhibit the RPS dynamical evolution when they engage in antibiotic production (see [4]). Actually, RPS games are a special class of spatially and temporally discrete dynamical systems characterized by local interactions. RPS game rules can produce a host of dynamical behaviors—ranging from the regular spatio-temporal evolution to the chaotic dynamics—by just designing a right kind of game rule. To some extent, a great deal of research of RPS games has been done to investigate the influence of diverse competitive mechanisms on the coexistence ratios of three species, see [5, 6, 7, 8, 9] and references therein. Roughly speaking, they address the questions not only in the framework of computer simulations, but also quantitative analyses based on the nonlinear partial differential equations. Not like the previous works, through the strict mathematical deduction, the purpose of this paper is devoted to providing a concise and preliminary discussion of symbolic dynamics of RPS game rules.

An interesting question is whether the intrinsic complexity of RPS games could be quantitatively analyzed in strict mathematical sense. We regard RPS games as a special extended class of cellular automata (CA), where each cell has three possible states (see [10, 11, 12, 13, 14, 15]). As one-dimensional CA, RPS game rules are conceived into an one-dimensional orthogonal grids of cells (aka squares).

In a series of fourteen papers (see [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]), L. O. Chua et al. provide a nonlinear dynamics perspective to Wolfram's empirical observations. For instance, the discrete ECA rules are described as scalar differential equations in the framework of continuous-time nonlinear dynamical system [16, 17]. They present the definitions of complexity index [17], global equivalence [18], $1/f$ power spectrum [19], fractal composition [20], time-reversal attractor [21], Isles of Eden [22, 23], etc. Under the periodic boundary condition, they provide several analytical tools such as Boolean cube [16, 17], characteristic function [18, 20], basin tree diagram [22, 23], Cobweb diagrams (aka Lameray diagrams) [19, 24], etc. In addition, it is necessary to note that some of their work is consistent with previous related studies. According to Wolfram's four classes of ECA rules (stability, periodicity, chaos and complex), Chua group ECA rules into six classes depending on the quantitative analysis of the orbits. These six classes are established as period-1, period-2, period-3, Bernoulli-shift, complex Bernoulli-shift and hyper Bernoulli-shift rules [24, 25, 26, 27, 28, 29].

Then, this article presents an accurate characterization of complex asymptotic dynamics of a particular RPS game rule. We are not to introduce the abundant evolution rules, such as reproduction and migration on a concrete probability in [8, 9]. As an illustration, the simple evolution rule is presented as follow: at first time step,

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§Manuscript received December 2015; revised February 2016.