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INDEPENDENCE ANALYSIS OF CHAOTIC SEQUENCES

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Abstract. The use of chaotic sequences in various domains has been the object of many research works. The main motivation is that, although a chaotic sequence is generated by a deterministic process, it can be considered as the result of a random process [1-3] and having almost all random features. In many works it has been assumed that chaotic sequences generated by a nonlinear map and beginning by two randomly chosen initial conditions are independent. In [18] we evoked this problem of independence by considering the Chi2 test used in statistics and we found two principal results, the first is that according to this test a randomly chosen couple of initial conditions can or not give two independent chaotic sequences; the second result is that the chance for a couple of two chaotic sequences to be independent depends on the considered map used to generate these sequences. In this paper we propose deeper study by considering Pearson coefficient and Kendall tau independence test; we analyze the statistics of these independence tests applied to chaotic sequences and for various chaotic maps.

Keywords. Chaotic sequences, independence, Chi2, Pearson Correlation, Kendal Tau.

1 Introduction

In the last years many searches have tackled with chaotic sequences; two aspects have been considered: theoretical and application aspects. In the theoretical aspect it has been shown that although a chaotic sequence is generated by a deterministic process it can be considered as the result of a random process [1-3]; statistical features of chaotic sequences such as auto and cross-correlation [4,5] and probability density function [6,7] have been the subject of recent researches. In the application aspect chaotic sequences have been used in telecommunications, cryptography and watermarking [8-14]; it has also been shown that this allows to improve the performance with respect to classical solutions. Especially, in spread spectrum communication systems, chaotic sequences have been considered as multiple access spreading codes; in most of the cases where the statistics of interferences between users was computed it has been supposed that these sequences are independent [9-10]. In some works the independence of variables generated by chaotic maps have been considered. In [16] bivariate chaotic maps $(x_{k+1}, y_{k+1}) = F(x_k, y_k)$ have been considered, the independence of variables x_k and y_k has been proven when F satisfies the equidistributivity property such as for the Arnold Cat map case. In [17] one dimensional chaotic maps $x_{k+1} = f(x_k)$, $x_k \in [0,1]$, with multiple monotonic branches have been considered; it has been shown that under some assumptions on f a discretized version u_k of x_k satisfies $E[u_k u_{k+s}] = E[u_k]E[u_{k+s}]$, where $E(X)$ is the mean of X; this shows that u_k and u_{k+s} are uncorrelated and not probabilistic independent. It has also been shown in this work that in some cases u_k and u_{k+s} are not uncorrelated and subsequently not independent.
In [18] and by using a new Lozi system some statistical properties have been considered in function of the bifurcations parameters of the system. Especially cross-correlation based independence analysis of individual couple of sequences has been considered; it has been seen that results depend on the bifurcation parameters. Otherwise a relation between geometric and statistical features has been highlighted in this work.
Without strong assumptions on chaotic maps it is difficult to prove theoretically the dependency or the independency of variables generated by these maps. The questions that should be asked is how we can measure the dependency of these variables and are there criteria that allow to decide if they are independent.

In [19], we tackled with this question of independence and analyzed Chi2 independence test of chaotic sequences; the main result we found is that we cannot draw an absolute conclusion; indeed for a randomly chosen couple of initial conditions the corresponding sequences could or not be independent; more over the chance for a couple of two chaotic sequences to be independent depends on the considered map used to generate these sequences. In statistics domain other independence tests are often used such as Pearson correlation coefficient, Kendall tau and Spearman rho, these tests can measure the dependence when sequences are not independent.
To explain the aim of this paper we consider a one dimensional map $f : I \mapsto I$ where I is an interval. Let x_0 and y_0 be two initial conditions and the two sequences $x = (x_0, x_1, ..., x_n)$ and $y = (y_0, y_1, ..., y_n)$ beginning by these initial conditions and $x_{k+1} = f(x_k)$, $\forall k = 0, 1, ..., n-1$. If we consider x (resp y) n realiza-

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