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ASYMPTOTIC STABILITY THEOREMS OF MILD SOLUTIONS OF 2D-STOCHASTIC NAVIER-STOKES EQUATIONS

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Abstract. In this paper, we consider exponential asymptotic stability of the mild solution of a stochastic 2D Navier-Stokes in a separable Hilbert space H. To be precise, we present results on stability with a general decay.

Keywords. Navier-Stokes; Mild solution; Stochastic; Stability.

1 Introduction

The mathematical theory of the Navier-Stokes equation is of fundamental importance to a deep understanding, prediction and control of turbulence in nature and in technological applications such as combustion dynamics and manufacturing processes. The incompressible Navier–Stokes equation has a long history (e.g., see [1], [2],[12], for some earlier studies) as a model to understand external random forces. In aeronautical applications random forcing of the Navier–Stokes equation models structural vibrations and, in atmospheric dynamics, unknown external forces such as sun heating and industrial pollution can be represented as random forces. In addition to the above reasons there is a mathematical reason for studying stochastic Navier–Stokes equation. A rigorous theory of the stochastic Navier–Stokes equation has been a subject of several papers. Several approaches have been proposed [1], [2], [12] ,[3] from the classic paper by Bensoussan and Temam [2], Bensoussan [1] to some more recent results e.g., by Cutland [5] and Caraballo and Xiaoging [7].

The exponential stability of flows is a very interesting and important problem in the theory of fluid dynamics, as the vast literature shows (see Temam [12], among others, and the references therein). In the deterministic case, it has been known for a long time that the solutions of 2D–Navier–Stokes equation tend to a stationary one (unique in fact) when time goes to infinity, see Temam [12]. On the other hand, another interesting question is to analyze the effects produced on a deterministic system by some stochastic or random disturbances appearing in the problem. In the paper of Caraballo Langa and Taniguchi [6], and Castillo-Fernandez [4], they consider the stochastic disturbances in the Itô sense, so the stabilization results proved should be interpreted in a suitable sense (see also [6]). These facts have motived the present work whose main objective is to show the exponentially asymptotic stability in the 2-th mean of the mild solution for a two dimensional Navier–Stokes equation under the presence of stochastic disturbances in the Itô sense. interpreted in a suitable sense. (Caraballo, Langa, Taniguchi [6] and Govindan [11]) to be precise, consider

$$\begin{aligned} dx = [\nu\Delta x - \langle x, \Delta\rangle x + \Delta p + F(x)]dt + G(x(t))dW(t),\\ t > 0; \quad divx = \nabla x = 0 \quad \text{in} \quad [0,\infty) \times D,\\ x = 0 \qquad \text{on} \qquad [0,\infty) \times \Gamma, \end{aligned} \tag{1}$$

$$x(0, x) = x_0(x) \quad \text{in} \quad x \in D. \tag{2}$$

where D is a regular open bounden domain of $\mathbb{R}^2$ with boundary Γ, x is the velocity field of the fluid, p the pressure, ν the kinematic viscosity, x_0 the initial velocity field, F the external force field, and $G(x(t))dW(t)$ the random field where $W(t)$ is an infinite dimensional Wiener process. The above classical form of the Navier–Stokes equation can be re–written in the following abstract form

$$\begin{aligned} &dx = [\nu Ax(t) + B(x(t)) + F(x(t))]dt + G(x(t))dW(t);\\ &x_0 = x(0). \end{aligned} \tag{3}$$

see section 2 for details. The format of the rest of this paper is as follows. In the second section, we give the preliminaries containing several definitions and a Lemma. In the third section we prove some results the exponential stability. Based on a basic robustness analysis, the criteria for exponential stability of the stochastic 2D–Navier–Stokes equation are obtained in Theorem (3.1). In the fourth section, using this result from the third section, we obtain almost sure asymptotic behavior of the mild solution of 2D–Navier–Stokes Equations.

2 Preliminaries

We begin reminding basic definitions and results about the Navier-Stokes equation. The stochastic Navier-Stokes

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