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SIGNALS AND SYSTEMS IN LEARNING AND MEMORY

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Abstract. An oscillation in the brain is characterized by four physical quantities; *amplitude, frequency, phase and entropy.* While the first three quantities correspond to its activity in environment where the background noise is at low levels, the last quantity reveals its state in noisy real world conditions. We explored models pertaining to the first case. As far as functionality of healthy brain is concerned, phase information is most crucial. We report existence of nonlinear oscillations which preserve phase in single neuron systems; modified Morris Lecar (M-L) and Hindmarsh Rose (H-R) models. The bifurcation analysis of both single neural systems is presented. The bifurcation scenarios suggest that these systems support interesting dynamical transitions in parameter ranges which are disjoint. Phase-coupled oscillations represented by limit cycles in the phase space of the system and synchronization analysis of the complex network of neuron models are believed to play a crucial role in information transport. The phase-coupled oscillations also plays important role in learning and memory processes. Time trajectories originating for different initial conditions lag behind each other while expected behavior is that these would overlap. The complex connections of network and coupling strength present significant results and which are very efficient to clinical medical research. .

Keywords. Spiking bursting. Morris-Lecar model. Hindmarsh-Rose model. Bifurcation. Phase-coupled oscillations. Network connections. Synchronization.

1 Introduction

Oscillatory brain activity shapes the functional architecture of the working brain. Neuronal networks displaying oscillations in different bands are selectively distributed in the brain. This activity is explored in neuronal models. A memory process involves perception of a sensory input which is similar in information content to already stored in neuronal cells. Basar et al. [1] have argued that sensory perception is the result of interplay between cognition and memory. Rai et al. [2] studied a model based on new kind of oscillations (phase coupled) in brain which carry amplitude, phase and time information. These phase-coupled oscillations were discovered in an extended version of a Morris-Lecar model [3] which is derived from the Hodgkin-Huxley model [4].

There are three stages of memory: sensory, short-term and long-term [5]. Sensory memory is classified into three categories: visual, auditory and olfactory. Short-term memory stores single or chunked items for 30 seconds without repetition. The reasoning process is an essential component of short-term memory. The learning process [6, 7] involves transfer of information from short to long term memory. Encoding happens while information is repeatedly processed in the hippocampus. In rapid eye movement (REM) [8, 9], memories are replayed and reinforced in the hippocampus. Permanent memory traces are stored where sensory inputs first occurred. They are connected through neuronal networks. Memories are stored in complex neuronal networks spread over the entire brain surface.

Hodgkin and Huxley [4] developed a model for generation of action potential in the axon of the squid in terms of time and voltage-dependent sodium and potassium conductance G_{Na}. is decided by three activation particles m and one inactivating particle h. The potassium conductance is governed by four activating particles, n. It was observed by Fitzhugh and Nagumo [10] independently that the membrane potential, $v(t)$ and sodium activation, $m(t)$ evolves on similar time-scales during action-potentials. On the other hand, sodium inactivation, $h(t)$ and potassium activation, $n(t)$ change on slower time-scales. Fitzhugh-Nagumo model does not serve as a description of dynamic behavior of realistic neurons; e. g., rapid firing of the neuron. Hindmarsh and Rose [11] modified the Fitzhugh-Nagumo model by replacing the linear function, $g(x)$ with a quadratic function. The model produces spikes when being stimulated by a positive current. It is possible to switch between the stable resting state and stable limit cycle by changing the applied current.

A biological neuron does not fire with a constant frequency, but the firing slows down in due course of time and eventually terminates. This is called firing frequency adaptation [12, 13]. This is achieved by adding an extra variable to the two dimensional system of Hindmarsh-Rose dynamical equations. This extra variable represents a slowly varying hyperpolarizing current. Suitable choic-

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