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A NEW APPROACH IN FRACTIONAL VARIATIONAL ITERATION METHOD TO SOLVE THE FRACTIONAL ORDER DIFFERENTIAL EQUATIONS

V.Mishra, *†

Abstract. The article investigates the solutions of differential equations with fractional order derivatives applying a modified fractional order variational iteration method. The Lagrangian multiplier in the proposed method is expanded in the form of fractional order Taylor series, which helps to achieve the solutions in approximate analytical form.

Keywords. Correction functional, Jumarie's fractional derivative, Mittag-Leffler function, Diffusion equation .

1 Introduction

General Lagrange multiplier method was proposed by Inokuti et al.[1] in 1978 for solving non-linear problems,with the help of which the variational iteration method (VIM) was proposed by Chinese scientist J.H.He([2]-[3])in 1998-99. VIM has been successively used by various authors to solve linear and nonlinear differential equations([2]-[10]). The reliability and simplicity of the method and the reduction in size of the computation gave the method a wide applicability. In 1998, He[2]first applied VIM to solve fractional order differential equation. In 2010, Wu and Lee[11]proposed fractional variational iteration method which is very much useful for solving fractional order differential equations.Wu [12] also used fractional variational iteration method to solve fractional order nonlinear differential equations.In 2011,Khan et al.[13] used fractional variational iteration method for fractional initial boundary value problems arising in applications of nonlinear science. VIM needs the establishment of correction functional, evaluation of Lagrange's multiplier and the initial approximation. The most challenging step is the calculation of Lagrange's multiplier. In 2010, Jafari and Alipoor[14] proposed a new method for calculating general Lagrange multiplier in the VIM. Jumarie[15] proposed a new modified Riemann-Liouville left derivative for fractional order derivative,which is not required to satisfy higher integer order derivative than α.It is also proposed that α-th derivative of a constant is zero, where α is any arbitrary real number.

*V.Mishra is with Department of Mathematical Sciences, IIT(BHU), Varanasi-221005, INDIA. E-mail: vivmis2010@gmail.com

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In the past century notable work is done in the area related to diffusion equations. An interesting behavior of these diffusion equations is that it generates the Brownian motion(BM).Many authors viz., Wazwaz[16],Sadighi and Gangi[17] etc.have used VIM to solve diffusion equations.Fractional diffusion equation is obtained from classical diffusion equation in mathematical physics by replacing the first order time derivative by fractional order derivative of order $\alpha(0 < \alpha < 1)$.It is known fact that anomalous diffusion is characterized by diffusion constant whose mean square displacement is calculated as $\langle X^2(t)\rangle \sim t^{\alpha}$.The phenomena of anomalous diffusion is usually divided into anomalous sub-diffusion for $0 < \alpha < 1$ and anomalous super-diffusion for $1 < \alpha < 2$ .The ordinary diffusion corresponds to $\alpha = 1$ and limiting case $\alpha = 2$ corresponds to the wave equation which is called “ballistic” diffusion. Wu [18] has solved time fractional diffusion equation in porous medium and also fractional order diffusion equation in the case of local verses nonlocal[19]. Das([20]-[21])used VIM to find the solutions of fractional diffusion equation of order α where $0 < \alpha \leq 1$ for various initial conditions involving power of x. Later, Vishal and Das[22] have solved the nonlinear fractional order diffusion equation with absorbent term and external force using Optimal homotopy analysis method .

Anomalous diffusion and relaxation behaviors are often described in terms of fractional order equations, and generalized kinetic and stochastic equations which are a class of non-Markovian stochastic processes. For example, fractional Brownian motion (FBM) is a very useful approach to anomalous diffusion. It represents a random process driven by so-called fractional Gaussian noise. FBM is closely related with the generalized Langevin equation (GLE) for a particle driven by fractional Gaussian noise. An alternative approach to anomalous diffusion is the continuous time random walk (CTRW) which is a random walk subordinated to a renewal process in which each random particle jump is preceded by a random waiting time. The CTRW theory generalizes the results of the standard random walk concept.

Ordinary linear and nonlinear differential equations of fractional order in which unknown function is operated