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COMPLEX DYNAMICS IN HYBRID TOTALISTIC CELLULAR AUTOMATA RULE 3, 13 AND 10

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Abstract. In this article, the dynamical behaviors of hybrid totalistic cellular automata (HTCAs) are researched from the viewpoint of symbolic dynamic. The HTCA(3,13,10) exhibits a wide range of traveling and stationary localisations, it generates a lot of gliders in hybrid mechanism. Using quantitative approach we get down to search an exhaustive gliders from the HTCA(3,13,10) and classify them. After classifying and coding of the newly discovered gliders, we provide a symbolic dynamics method to analyze the chaotic dynamics properties of the glider. As addition, it is proved that the glider can be expressed as a particular subsystem with complicated dynamical properties, such as topologically mixing and positive topological entropy, so that it is chaotic in the sense of both Li-Yorke and Devaney in corresponding subsystems.

Keywords. cellular automata, Hybrid mechanism, Symbolic dynamics, Glider, Chaos, Topologically mixing, Topological entropy.

1 Introduction

Cellular automata (CAs), which were introduced by John von Neumann from the late 1940s to the early 1950s, are a class of spatially and temporally discrete, deterministic mathematical systems with large degree of freedom characterized by local interactions and an inherently parallel form of evolution [1-3]. Some mathematical theory of cellular automata was developed later by Hedlund in the context of symbolic dynamics [2]. Most of the research on cellular automata as dynamical systems followed by Wolfram's works on dynamical and computational aspects of CAs [2-6]. At the same time, he concentrated on the analysis of dynamical systems and studied in detail of elementary cellular automata and classified the 256 rules informally into four classes in his works by using dynamical concepts like stability, periodicity, chaos and complex [3,7,12-18]. Moreover, Wolfram proposed cellular automata as models for physical system exhibiting complex or chaotic behaviors. As a matter of fact, rules in Wolfram's class IV, which includes the Game of Life and related rules, produce gliders in their evolution, thus post a fruitful subject for investigations not only in terms of complex systems and self-organization but also for dynamics of non-classical computation. According to Wolfram's four classes of ECA rules, Chua et. al divided ECA rules into six classes depending on the quantitative analysis of the orbits. These six classes are period-1, period-2, period-3, Bernoulli-shift, complex Bernoulli-shift and hyper Bernoulli-shift rules respectively [11-14].

Although CAs have simple structures, they possess rich dynamical behaviors. In addition, CAs are mathematical models, there exhibits different kinds of varieties and complexities through projecting the different local rules. The local rules describe precisely how a given cell should change states, depending on its current state and the states of its neighbors as well. Particularly, 1D CAs with simple local rules can exhibit complex dynamical behaviors and have comprehensive applications in many scientific fields[8]. For example, we can apply CAs to simulate a whole hierarchy of structures and phenomena in some cases. The research of 1D CAs has caused the extensive concern from many scientists over the years, the reason behind it is that their simplicity, and they have enormous potential in modeling complex systems.

Under the condition of hybrid mechanism, the original dynamical properties of these basic rules are changed, some of them are simpler than the original one, and some of them are more complex than the original one. Here, the totalistic cellular automata(TCAs) rules depend on the total of the values of the cells in three inputs. The evolution of an 1D TCA can completely be described by a table specifying the state of each given cell will have in the next generation, which is based on the average value of the three cells consisting of the value of the cell to its left, itself, and its right. There are 2^4 different TCA rules. Each of them corresponding to the only spatio-temporal pattern. The spatio-temporal patterns of all TCA rules are displayed in Fig. 1. For one 1D CAs, when the evolution of every single cell is not dependent on the unique global function, it is called uniform, otherwise it is called hybrid, i.e. HTCAs. Although HTCAs are endowed with simple hybrid rules and evolve on the same square tile structures, the evolution of HTCAs may exhibit a wide range of traveling and stationary localisations [19].

In this paper, hybrid cellular automata (HCAs) rule is

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