Journal of Nonlinear Systems and Applications (2017) 1–7

http://www.watam.org/JNSA/

COMPLEX DYNAMICS IN HYBRID TOTALISTIC CELLULAR AUTOMATA RULE 2 AND 39

Mengmeng Meng, Fangyue Chen, Bo Chen and Lingna Zhao *†

Abstract. Rule 2 and 39 are considered to have simple Bernoulli-shift dynamics, but the couple of rules yield numerous complex dynamical behaviors under the function of hybrid and totalistic mechanism. In this paper, complex dynamics of HTCA(2,39) is discussed from a symbolic dynamics perspective. It is found that there exist three types of glider behaviors in its three particular subsets. Furthermore, we prove that HTCA(2,39) is chaotic in the sense of both Li-Yorke and Devaney in its three particular subsystems respectively by demonstrating that they are topologically mixing, topologically transitive and have positive topological entropy. Finally, it is worth mentioning that the method presented in this paper is also applicable to other HTCAs therein.
Keywords. Hybrid totalistic cellular automata, symbolic dynamics, chaos, topologically mixing, topological entropy, glider

1 Introduction

John von Neumann introduced cellular automata (CAs) formally in 1951, which are a class of spatially and temporally discrete, deterministic mathematical systems. The structures of CAs have large degrees of freedom characterized by local interactions and an inherently parallel form of evolution [31]. Against a background of symbolic dynamics, Hedlund probed mathematical theory of CAs in 1969 [21]. In the early 1980s, Wolfram focused on the analysis of dynamical systems and studied in detail of 1D CA rules, which are classified informally into four classes [32, 33]. Starting from random initial configurations, class I rules evolve to homogeneous final states, class II rules yield separated periodic patterns, class III rules show chaotic behaviour, and yield aperiodic structures, and class IV rules exhibit complicated propagating and localized structures.

Besides, a glider is a compact group of non-quiescent states which have a periodic structure moving in time [36, 4, 23]. In class IV, glider phenomena emerged in the evolution of the rules. The property of the rules provides an attractive topic for investigates not only in terms of complex systems but also for non-classical computation. Noticeably, the glider phenomena have consequently become the focus of many particular studies [36, 23, 24, 25, 22]. Based on Wolfram's work, Chua et al. provide a nonlinear dynamics perspective from the viewpoint of mathematical analysis [34, 10, 11, 12, 13, 14].

In the early 1980s, Wolfram has introduced the concept of a totalistic CA (TCA) [32, 35]. In this model, the state of an automaton is considered as a nonnegative integer and the state transition function depends only on the sum of the states of the cell's neighborhood, the sum includes the cell's own state. In the light of an abundant statistical evaluation, he affirmed that 1D TCAs may be computation-universal [35]. Then, Dan Gordon proved his conjecture by presenting that TCAs can simulate any Turing machine in linear time [20], it is well known that CAs can simulate an Turing machine [1, 6, 17, 26, 27]. There are some researches of TCAs on neural network, physics and so on[2, 3, 19].

For an 1D CA, when all cell employ different rules for determining its next state, the CA is called hybrid, i.e. hybrid CA (HCA). This is different from the uniform CAs promoted by, e.g., Conway [18] and Wolfram [34], in which all cells use the same rule. Besides, [8] discussed the complex dynamics of HCA(77,168) and HCA(28,33) by means of symbolic dynamics, there will also be glider collisions from the evolution of HCA(168,133) [9]. These articles are very helpful for writing this paper. There are a lot of researches on HCAs which have been applied in cryptographically secure, see [30, 28, 29] and references therein.

In this paper, we will discuss the complex dynamics in HTCA 2 and 39 (denoted by HTCA(2,39)) with five neighbors based on the properties of HCAs and TCAs. Thus, HTCA(2,39) is composed of TCA rule 2 and TCA rule 39, which is specified to obey rule 2 at odd sites of the cell array, and obey rule 39 at even sites of the cell array. Although HTCAs are endowed with simple hybrid and totalistic rules, evolve on the same square tile structures, the evolution of HTCAs may exhibit rich dynamical behavior through local interactions. So, it can be conjectured that the dynamical behaviors of the CAs might be changed from simple to complex by introducing

*Mengmeng Meng, Fangyue Chen, and Lingna Zhao are with Department of Mathematics, School of Science, Hangzhou Dianzi University, China. Bo Chen is with Department of Electronic Engineering, School of Information Science and Technology, Fudan University, China. E-mails: 1543744046@qq.com, fychen@hdu.edu.cn, chenbo4068922@126.com, 673641666@qq.com

†Manuscript received October 27, 2016; revised November 12, 2016.