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COMPLEX MACROSCOPIC BEHAVIOR OF PHASE OSCILLATORS IN PRESENCE OF HIGHER HARMONIC INTERACTIONS

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Abstract. The collective motion of Kuramoto[1] like model in presence of higher harmonic interaction is studied in detail. It is demonstrated that in the thermodynamic limit , the Ott-Atonsen[2] prescription is still useful in extracting the dynamics of the order parameter in contradiction to the earlier observation of Chiba[3] and others. Our analysis clearly displays a hysteresis type behavior in the order parameter and it also indicate to the formation of more than one cluster. It is observed that through the basic bifurcation pattern remains the same yet some new structure appears near sub and super critical scenario.

1 Introduction

Collective synchronization is a form of self organization in time which is an outcome of interaction between a very large population of heterogeneous oscillators. Such phenomena is observed in large variety of systems that range from biology to chemistry, physics and engineering. In all these events the mutual interaction succeed in entraining the rhythms of such a heterogenous ensemble of self sustained oscillators. Example of such phenomena are synchronization ed flashing of fireflies[4]., cardiac pacemaker cells[5], walker induced oscillations of the Millennium bridge[6], josephson junction circuits[7], neural processing[8] and chemical oscillations[1]. Kuramoto showed that the evolution of phase in an ensemble of N weakly coupled oscillators obeys an equation of the form,

$$\dot{\theta}_n = \omega_n + \sum H_{mn}(\theta_m - \theta_n) \tag{1}$$

where θ_n is the phase, ω_n is the int6rinsic frequency, and H_{nm} is a 2π periodic function that describes the coupling between the nth and mth oscillators. When such oscillators represent limit cycles arising from a Hopf bifurcation their coupling is generically sinusoidal leading to

$$H_{nm}(\theta) = \frac{K_{nm}}{N} sin(\theta) \tag{2}$$

for uniform coupling $K_{nm} = K$ and one gets the basic Kuramoto model.

In recent times generalization of Kuramoto model has becomes an important are of research. People have tried to analyze the effect of non-sinousoidal coupling, the effect of network topology[9], external forcing[10], time dependent coupling[11] and noise[12]. On the other hand long Daido[13] attempted to analyze a case when $H(\theta)$ was a general function of the form,

$$H(\theta) = \sum \{a_k \sin(k\theta) + b_k \cos(k\theta)\} \tag{3}$$

but only studied the stationary situation, but not the ensuing dynamics. Here in this communication we have considered the usual Kuramoto model in presence of higher harmonic tem $K \sin(2\theta)$. It is observed that the order parameter dynamics changes in a significant way. In this connection it may be mentioned that a Kuramoto like situation where the basic term $K \sin(\theta)$ is absent but interaction is solely governed by $\sin(\theta)$ term was studied by Skardal et.al[14]. Our analysis reveals very interesting new behaviour of the order parameter. Important feature being the formation of more than one cluster and the existence of hysteresis. In this connection it will not be out of place to mention that a similar problem was attempted by Kori and Kuramoto long back[15], whose analysis was partly phenomenological and partly numerical. It may also be noted that situation with higher harmonics, Chiba[3] made the assertion that perhaps the reduction technique of Ott-Atonsen[2] will not be useful for the higher harmonic dynamics. On the otherhand our analysis clearly shows that the Kuramoto model in presence of higher harmonic interaction can very well be treated by the above said reduction mechanism and the behaviors of the order parameter be extracted. Of course we have analyzed the associated bifurcation problem.

2 Formulation

Consider N phase oscillators equally spaced on the one dimensional domain. The state of the ith oscillator at time t is $\theta_i(t)$. The strength of coupling between any pair

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