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S-ASYMPTOTICALLY ω-PERIODIC SOLUTION FOR A NONLINEAR DIFFERENTIAL EQUATION WITH PIECEWISE CONSTANT ARGUMENT VIA S-ASYMPTOTICALLY ω-PERIODIC FUNCTIONS IN THE STEPANOV SENSE

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Abstract. In this paper, we show the existence of a function which is not S-asymptotically ω-periodic, but which is S-asymptotically ω-periodic in the Stepanov sense. We give sufficient conditions for the existence and uniqueness of S-asymptotically ω-periodic solutions for a nonautonomous differential equation with piecewise constant argument in a Banach space when ω is an integer. This is done using the Banach fixed point Theorem. An example involving the heat operator is discussed as an illustration of the theory.

Keywords. S-asymptotically ω-periodic functions, differential equations with piecewise constant argument, evolutionnary process.

1 Introduction

In this paper, we study the existence and uniqueness of S-asymptotically ω-periodic solution of the following differential equation with piecewise constant argument

$$\begin{cases} x'(t) = A(t)x(t) + f(t, x([t])), \\ x(0) = c_0, \end{cases} \tag{1}$$

where $\mathbb{X}$ is a banach space, $c_0 \in \mathbb{X}$, $[\cdot]$ is the largest integer function, f is a continuous function on $\mathbb{R}^+ \times \mathbb{X}$ and $A(t)$ generates an exponentially stable evolutionnary process in $\mathbb{X}$. The study of differential equations with piecewise constant argument (EPCA) is an important subject because these equations have the structure of continuous dynamical systems in intervals of unit length. Therefore they combine the properties of both differential and difference equations. There have been many papers studying EPCA, see for instance [14], [15], [16], [17], [18] and the references therein.

Recently, the concept of S-asymptotically ω-periodic function has been introduced in the litterature by Henríquez, Pierri and Táboas in [8], [9]. In [1], the authors studied properties of S-asymptotically ω-periodic function taking values in Banach spaces including a theorem of composition. They applied the results obtained in order to study the existence and uniqueness of S-asymptotically ω-periodic mild solution to a nonautonomous semilinear differential equation. In [22], the authors established some sufficient conditions about the existence and uniquenes of S-asymptotically ω-periodic solutions to a fractionnal integro-differential equation by applying fixed point theorem combined with sectorial operator, where the nonlinear pertubation term f is a Lipschitz and non-Lipschitz case. In [2], the authors prove the existence and uniqueness of mild solution to some functional differential equations with infinite delay in Banach spaces which approach almost automorphic function ([6], [11]) at infinity and discuss also the existence of S-asymptotically ω-periodic mild solutions. In [20], the author discussed about the existence of S-asymptotically ω-periodic mild solution of semilinear fractionnal integro-differential equations in Banach space, where the nonlinear pertubation is S-asymptotically ω-periodic or S-asymptotically ω-periodic in the Stepanov sense ([10], [20], [21]). The reader may also consult [3], [4], [5], [7], [12] in order to obtain more knowledge about S-asymptotically ω-periodic functions. Motivated by [1] and [7], we will show the existence and uniqueness of S-asymptotically ω-periodic solution for (1) where the nonlinear pertubation term f is a S-asymptotically ω-periodic function in the Stepanov sense. The work has four sections. In the next section, we recall some properties about S-asymptotically ω-periodic functions. We study also qualitative properties of S-asymptotically ω-periodic functions in the Stepanov sense. In particular, we will show the existence of functions which are not S-asymptotically ω-periodic but which are S-asymptotically ω-periodic in the Stepanov sense. In section 3, we study the existence and uniquenes of S-asymptotically ω-periodic mild solutions for (1) considering S-asymptotically ω-periodic functions in the Stepanov sense. In section 4, we deal with the existence and uniqueness of S-asymptotically ω-periodic solution for a partial differential equation.

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