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EXISTENCE AND POSITIVITY OF A GLOBAL SOLUTION IN A SPATIO-TEMPORAL MODEL OF THE CHIKUNGUNYA DISEASE

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Abstract. In this paper, we study a chikungunya epidemic transmission model which describes an epidemic disease transmitted by Aedes mosquitoes. This model includes the spatial mobility of humans since it is probably a factor that has influenced the re-emergence of several diseases. Suppose that the spatial mobility of humans is random and described as Brownian random motion. For this reason, we introduce diffusion terms in the temporal model given in [9]. Since the displacement of mosquitoes is limited to a few meters, compared with humans, one can ignore mosquitoes mobility. Therefore, the complete model is composed of a reaction-diffusion system coupled with ordinary differential equations (ODEs). In this paper, we prove the existence and uniqueness, the positivity and boundedness of the global solution for the model and give some numerical simulations.

Keywords. Nonlinear model; Epidemiologic model; Chikungunya virus; Human mobility; Reaction-diffusion system.

1 Introduction

For many decades, our societies have been confronted to recurrent epidemiological diseases due to the mobility of humans, the adaptation of their vectors, the virus itself and the environment changes. Global health authorities are now strongly engaged in the control of these diseases and describing their spread has become a major issue for predicting their evolution and controlling their outbreak. In order to better apprehend vector-borne diseases, mathematical models have been developed and studied since the 20th century. For example, models for the transmission of the diseases transmitted by the the mosquito *Aedes albopictus* [15] have been proposed by [3] for the dengue and [2, 9] for the chikungunya. However, these first models do not consider space.

The description of the spread of such diseases can be done by epidemiological models, usually derived from the classical SIR models. These compartmental models consist in structuring the population in susceptible, infected and recovered individuals. Assuming that the spatial mobility is random and is described as Brownian random motion, the authors in [12] have proposed to add diffusion terms in system of ODEs to consider the spread of diseases. This modeling way is often used in order to take into account the spread of populations [1, 8].

In this paper, we focus on the chikungunya disease whose particularity is to reemerge regularly since the beginning of the 21th century. Until 2000, this virus was confined to African countries. However, because of the global warming and the development of transports, an unprecedented epidemic was observed in the Réunion island (a French island in the Indian Ocean) in 2005-2006 where one third of the total population was infected. The maximum number was reached in February 2006 with 40 000 infected. The chikungunya epidemic affected Europe for the first time in 2007 from Italy. It was observed that the vector of this epidemic had developed capabilities to adapt to non-tropical regions. In 2014, this epidemic spread to the whole of the Caribbean, the countries of America. Hundreds of cases of the islands of Oceania also exported to Europe and elsewhere in the world.

In [9], a SI-SIR model that took into account the biological life cycle of mosquitoes and described the spread of the virus to humans was proposed. The study of the parameters for this model was done by [16]. However, this model does not consider the spatial mobility of humans, which is a factor that probably has influenced the re-emergence of several epidemics. In [11], a spatio-temporal model in the form of metapopulation model was proposed. This model considers a graph where each node represents a real-world habitat. At each node, the SI-SIR model is considered. In order to model the mobility of humans, each node is connected to some neighboring nodes. This spatio-temporal model can be considered as a discrete model of spatial variables. In this paper, we present a continuous model of space and time variables, which is also based on the SI-SIR model proposed in [9]. We assume that the mosquito's displacement is limited to a few meters and therefore can be neglected compared to humans. For modeling the human mobility, the idea of [12] is taken again.

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